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NATURAL PHILOSOPHY.

A FIRST BOOK

OF

NATURAL PHILOSOPHY;

OR,

AN INTRODUCTION

TO THE STUDY OF

STATICS, DYNAMICS, HYDROSTATICS, AND OPTICS.

WITH NUMEROUS EXAMPLES.

BY

SAMUEL NEWTH, M.A.,
FELLOW OF UNIVERSITY COLLEGE, LONDON.

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PREFACE.

THE first edition of my "Elements of Statics, &c." being now exhausted, I avail myself of the opportunity afforded by the preparation of a second edition for the press, to publish separately, in the present work, those portions which include the subjects required at the Matriculation Examination of the University of London. The demonstrations have been carefully adapted to the requirements of those whose Mathematical knowledge does not extend beyond the First Book of Euclid, and the easier cases of Simple Equations. In a few instances, where the demonstrations given in the "Elements" presented some perplexity to the beginner, experimental proofs have been substituted. Examples in illustration are given with all the more important propositions, and numerous other examples are added for exercise. I have endeavoured hereby to supply a work which should be generally useful as a First Book of Natural Philosophy. Some experience in teaching has confirmed the opinion, that the junior pupil may, with a twofold benefit, be early introduced to Natural Philosophy, as a branch of his Mathematical studies. The interesting applications which this study supplies of the processes and results of pure Mathematics greatly assist the beginner in thoroughly mastering these; and his early familiarity with Mechanical principles, and some at least of their practical applications, enables him afterwards to prosecute their more formal study at a considerable advantage.

SAMUEL NEWTH.

Plymouth, *May*, 1854.

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NATURAL PHILOSOPHY.

CHAPTER I.

DEFINITIONS AND PRINCIPLES.

1. WHATEVER is capable of *producing* motion in a body, or any *change* in the motion of a body, is termed *force.*

In other words, force is the name we employ to express that unknown cause which, under any circumstances, can produce a change in the state, whether of rest or motion, of any material body.

Whatever causes a change in the motion of a body must be regarded as of like nature with that which produces motion, and hence the same term (force) is applied to both, even although there are some forces which, while they are able to change the motion of a body, can never produce it. Such, for example, are friction and resistances of all kinds. Forces of this nature can, it is evident, never act alone, for some other force must be present in order to pròduce the motion which they change, and hence if but one force act upon a body, it must be one capable of *producing* motion.

2. If one force only act upon a body, motion must necessarily follow; but when two or more forces act

upon the same body, their united effect *may* be such that no motion ensues. Whenever this is the case, the forces are said to be *in equilibrium*.

3. That branch of mechanics which investigates the relations which exist between forces in equilibrium is termed Statics; and that which investigates the effects of forces not in equilibrium, but producing motion, is termed Dynamics.

4. Whenever motion is prevented by muscular effort, as for instance, when a weight is held in the hand and so prevented from falling to the ground, or when an elastic cord is stretched and its rebound prevented, a sensation is produced which we call *pressure*. But, just as in several other cases, the same word is used both for the sensation and its cause—*e.g.* sound, smell, taste—so is the term pressure applied also to the force whose resulting motion has been thus prevented. In this sense, then, pressure is force considered as the cause of the sensation which is felt when motion is prevented by muscular effort. It is with this meaning that the term is commonly employed in mechanics, although extended to all forces when the motion they are capable of producing is in any way prevented, whether it be by muscular action or not. Thus, whether a weight be held in the hand or rest upon a table, in either case it is said to exert a pressure, and a pressure of the same amount.

The commonest case of pressure is weight, and this supplies the most convenient standard of reference by which to compare different pressures. By means of weight other pressures may in most cases be easily measured. Thus, when an elastic cord is held stretched by the hand, the pressure it exerts may be compared with others by determining what weight will keep the string stretched to the same degree.

Since all questions considered in Statics refer to forces in equilibrium, all statical forces may be denominated pressures, and consequently may be measured by weight.

5. Forces may differ from each other, not only in magnitude, but also in direction, and hence may be conveniently represented by straight lines; the direction of the lines representing the direction of the forces, and the lengths of the lines, each being measured by the same scale, the magnitudes of the forces.

Two forces will be called *concurrent*, if they both push towards or from the same point. In the case of parallel forces, this point is supposed to lie at an infinite distance.

6. Force, when transmitted by means of a cord, is sometimes spoken of as the *tension* in the cord.

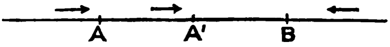
7. PRINCIPLE I. *If two forces, equal in magnitude and opposite in direction, act upon a material particle, they are in equilibrium: and conversely, if two forces, acting upon a material particle, are in equilibrium, they are equal and opposite.*

This is the simplest possible case of equilibrium, and is assumed as an axiom.

8. PRINCIPLE II. *When forces are in equilibrium, the equilibrium will not be disturbed by the introduction or removal of any number of forces that are themselves also in equilibrium.*

Forces in equilibrium can neither produce nor change motion; and, consequently, the introduction of such a set of forces will not cause motion; nor, on the other hand, can motion ensue from the removal of such a set of forces: for, if it could, that motion must have been checked by their presence, or forces in equilibrium would have destroyed motion, which they cannot do.

9. *Transmissibility of force.*—A force acting upon a rigid body may be supposed to act at any point, within the body, in the line of its direction. Thus, if a force of any magnitude P act upon a rigid body at A, and along the line AB in the direction of the arrow, we may suppose P to be applied at any other point A′ in the line AB within the body. For at A′ and B introduce two equal and opposite forces of the same magnitude as P. At A and B we have two equal and opposite forces. These forces are (Principle I.) in equilibrium, and may therefore (Principle II.) be removed. There will then remain only a force P at A′ of the same magnitude as the original force, and acting in the same direction.



10. The single force, which represents the combined effect of several forces, is termed their *resultant;* relatively to the resultant, these several forces are termed *components* or *component forces. Composition of forces* takes place when two or more forces are replaced by a single force equivalent to them, that is, when the resultant is substituted for its components. *Resolution*

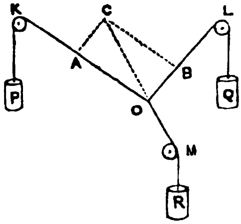
of forces takes place when a single force is replaced by two or more forces equivalent to it, that is, when the components are substituted for their resultant.

11. It is evident, that if two concurrent forces act along the same line their resultant is their sum; and that, when two non-concurrent forces act in the same line, the resultant is their difference. These are respectively the greatest and least values of the resultant of two forces. When the two components are not acting both together, or both opposed to each other, but inclined to each other at any angle, the resultant will have some value intermediate between these extremes. It will presently be seen that, as we should expect beforehand, the smaller the angle between the components, the more nearly will the resultant approach to the greatest possible value, and the larger the angle between the components the more nearly will it approach its least possible value. The exact value of the resultant of two forces is found by aid of the principle commonly termed the *parallelogram of forces.*

12. If any number of forces be in equilibrium, any one force must be equal in magnitude and opposite in direction to the resultant of the remaining forces. For if, instead of the remaining forces, we substitute their resultant, we shall have but two forces; namely, this resultant and the force in question, and these are in equilibrium, and therefore, by Principle I. they are equal and opposite.

13. PRINCIPLE III. (PARALLELOGRAM OF FORCES.) *If two concurrent forces acting upon a point are represented in magnitude and direction by the two sides of a*

parallelogram, then will their resultant also be represented in magnitude and direction by the diagonal drawn through the given point.



This proposition may be proved experimentally in the following manner. Let two cords, bearing the weights P and Q, pass over the fixed pulleys K and L, and be connected together at O. Let O, for convenience sake, be temporarily fastened by a pin. Along OK mark off OA, representing the weight P on any scale; that is, take as many divisions of the scale as there are pounds or ounces at P. Along OL mark off OB, representing Q on the same scale. Complete the parallelogram AOBC and draw the diagonal CO. Let a third string be attached to O, and pass in the direction of CO, produced over a pulley M, and a weight R be attached to it of the magnitude represented by the line CO. Then, if the pin be removed and O set at liberty, it will be always found that O remains at rest; that is, the forces P, Q, and R, acting upon it are equilibrium. But if three forces be in equilibrium, any one must be equal and opposite the resultant of the other two. Consequently, R must be equal and opposite to the resultant of P and Q. Hence the diagonal CO represents the magnitude and direction of the resultant of the forces P and Q, represented respectively by the sides AO and BO.

14. Since the components may be substituted for their resultant, their effects being equivalent, it follows from the preceding, that for any force we may substi-

tute two others, whose magnitude and direction are represented by the sides of any parallelogram, of which the line representing the given force forms the diagonal. And since an infinite number of parallelograms can be drawn, having a given line for their diagonal, any force can be resolved into two others, in an infinite number of ways.

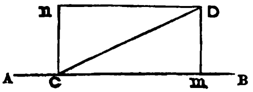
15. *If three forces* act upon a point, and their resultant be required, find the resultant of any two of them; the composition of this resultant, with the third force, will give the resultant of the three given forces. In like manner, the resultant of any number of forces acting upon a point may be found.

16. By the aid of the parallelogram of forces, the resultant of two forces can always be easily determined geometrically, and its numerical magnitude thence found by applying to the diagonal the graduated scale used in marking off the sides of the parallelogram.

The magnitude of the resultant may be also in some cases computed arithmetically; that is, whenever the length of the diagonal can be so computed. The following are examples.

Ex. 1. To find the resultant when forces of 36 and 48 lbs. act upon a point at right angles to each other.

Let C*n* and C*m* represent the two forces on any scale, then, completing the rectangle, CD will represent the resultant on the same scale. But CD is the hypo-

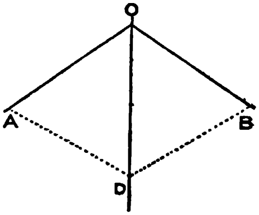


tenuse of a right-angled triangle whose sides are 36 and 48; ∴ Euclid, b. i. prop. 47.

$$\begin{aligned} CD^2 &= 36^2 + 48^2 \\ \therefore\ CD &= \sqrt{(1296 + 2304)} \\ &= 60. \end{aligned}$$

Ex. 2. Two equal forces act upon a point at an angle of 120°, to find their resultant.

Let the equal lines AO, BO, making the angle AOB = 120° represent the forces. Complete the parallelogram, then OD represents the resultant. But the angle AOB is 120°; ∴ the angle at A is 60°; and since the remaining angles of the triangle AOD are equal to each other, they each = 60°. Therefore the triangle AOD is equiangular, and, consequently, equilateral. Hence OD = OA, or, the resultant is equal to either of the components.



Ex. 3. Two forces, each of 10 lbs. act upon a point at an angle of 60°, to find the resultant.

Let AO, AD, in the preceding figure, represent the two forces. The diagonal drawn from A to B will represent the resultant. This diagonal will cut OD at right angles, and will also bisect it; call the point of intersection, E. Then the triangle AEO is half an equilateral triangle, and therefore OE = ½ OA = 5.

$$\begin{aligned} AE^2 &= AO^2 - OE^2 \\ &= 100 - 25 \\ AE &= \sqrt{75} = 8{\cdot}66 \\ AB &= 2AE = 17{\cdot}32. \end{aligned}$$

Ex. 4. The resultant of two equal forces acting upon a point at an angle of 60° is 18 lbs. to find the magnitude of each component. Using the same figure as before,

$$
\begin{aligned}
AE &= \tfrac{1}{2}\,AB = 9 \\
AO^2 &= AE^2 + OE^2 \\
&= 9^2 + (\tfrac{1}{2}\,AO)^2 \\
\tfrac{3}{4}\,.\,AO^2 &= 9^2 \\
AO &= \sqrt{108} = 10{\cdot}39.
\end{aligned}
$$

EXAMPLES.

1. Find the resultant when forces of 50 and 120 lbs. act upon a point at right angles to each other. Ans. 130 lbs.

2. Find the resultant when forces of 16 and 63 lbs. act upon a point at right angles to each other. Ans. 65 lbs.

3. Two forces, each of 100 lbs. act upon a point at an angle of 60°, find their resultant. Ans. 173·2 lbs.

4. Two forces, each of 100 lbs. act upon a point at an angle of 120°, find their resultant. Let AO and BO, in the figure on page 8, represent the two forces. Ans. 100 lbs.

5. The resultant of two forces acting upon a point at right angles is 145 lbs. and one of the components is 24, find the other component. Ans. 143 lbs.

6. The resultant of two equal forces acting upon a point at an angle of 60° is 12 lbs. find the magnitude of each component. Ans. 6·92.

7. The resultant of two forces acting at right angles is 20 lbs. and makes an angle of 30° with one of the components, find the two components. Ans. 10 and 17·32.

8. Find the resultant of three equal forces lying in

one plane, when the first and third each act at an angle of 60° with the second.

9. Find the resultant of three forces of 10 lbs. each, lying in one plane, when the first and third each act at an angle of 30° with the second. Ans. 27·32.

10. Find the resultant of three forces, 10, 10, and 20, respectively, lying in one plane, when the first and second act at an angle of 90°, and the second and third at an angle of 45°. Ans. 24·49.

11. Two forces, 603 and 2412 respectively, have a resultant 2613, at what angle do the components act? Ans. 90°.

12. Shew, by a geometrical construction, that if four equal forces lying in a plane act upon a point in such a way that the angle between the first and second, that between the second and third, and that between the third and fourth, are each equal to 72°, the resultant shall be equal to either of the components.

13. Let A, B, and C be three pegs in a vertical wall, over which passes a string with an equal weight attached at each end; determine the amount and direction of the pressure on each peg when ABC forms an isosceles triangle, whose base to horizontal, and whose vertical angle is 120°.*

17. PRINCIPLE IV. (TRIANGLE OF FORCES.) *If*

---

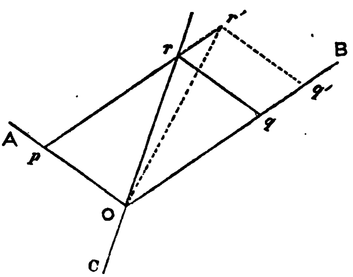
* Before reading the next Section, the following geometrical propositions should be proved:—

1. The angle contained by parallels to two lines is equal to the angle contained by the lines themselves.

2. The angle contained by the perpendiculars to two lines is equal to the angle contained by the lines themselves.

3. If two triangles have one side in each equal, and the sides of the one severally parallel or severally perpendicular to the sides of the other, the two triangles shall be equal in every respect.

three forces acting upon a point be in equilibrium, and if any triangle be drawn, whose sides are severally parallel or severally perpendicular to their directions, the forces will be to one another as the sides of the triangle: and conversely, if the three forces be as the sides of the triangle they will be in equilibrium.



Let P, Q, and R be three forces in equilibrium acting upon the point O, and along the lines OA, OB, and OC respectively. Let any triangle be supposed, whose sides are severally parallel, or severally perpendicular to the directions of P, Q, and R. In CO produced, cut off O*r* equal to that side of this triangle which is parallel or perpendicular to the direction of R. Draw *rp* parallel to OB, and *rq* parallel to OA, then the triangle O*pr* is equal to the given triangle.

Since the three forces are in equilibrium, the resultant of any two of these must be equal and opposite to the third force; hence the resultant of P and Q must fall along O*r*.

Then if O*p* be taken to represent the magnitude of the force P, O*q* must represent the magnitude of the force Q; for if the magnitude of Q were represented by any other length O*q'*, then by the parallelogram of forces, the resultant of P and Q would fall along O*r'*. Then since the forces P and Q are represented in magnitude by the lines O*p* and O*q*, their resultant is (Principle III.) represented in magnitude by O*r*. But the force R is equal to the resultant of the forces P and Q.

Therefore P, Q, and R are represented by the lines Op, Oq, Or, and therefore, since $pr = Oq$, by the three sides of the triangle Opr, and, consequently, by the three sides of the given triangle.

Conversely. Let P, Q, and R be as the sides of the supposed triangle, they shall be in equilibrium. For, as already shewn, Op, Oq, and Or are equal severally to the sides of this triangle. But if the forces P and Q are represented in magnitude and direction by the lines Op and Oq, then their resultant is represented by Or, and, consequently, is equal and opposite to R; there is, therefore, equilibrium.

18. *Ex.* 1. If a ball, supposed to be without weight, slide along a perfectly smooth rod, and a force of 10 lbs. act on the ball at an angle of 30° with the rod, to find the force which must be applied in the direction of the rod to keep the ball at rest.

Let AB (fig. art. 16) be the rod, and DC the given force of 10 lbs. acting upon the ball at C. Let the required force act upon C, there will then be three forces in equilibrium; viz., the given force, the resistance of the rod which acts at right angles to the rod, and the required force. Draw Dm at right angles to AB. The three sides of this triangle are severally parallel to the three forces; viz.—

CD parallel to the given force.
Dm „ „ resistance.
Cm „ „ required force.

Therefore, by the triangle of forces, these three forces are severally represented by these three lines. Hence

Cm represents the force required. But CD = 10, and since the triangle CDm is half an equilateral triangle Dm = 5.

$$\begin{aligned} Cm^2 &= CD^2 - Dm^2 = 100 - 25, \\ Cm &= \sqrt{75} = 8{\cdot}66, \end{aligned}$$

$\therefore$ the required force = 8·66.

Ex. 2. To find the horizontal and vertical pressures, when a force of 20 lbs. acts in a direction making an angle of 60° with the vertical line.

Let CD (fig. art. 16) represent the given force. Let forces equal and opposite to the horizontal pressures act at C. These forces will evidently be in equilibrium. The sides of the triangle CDm are severally parallel to these three forces. Hence Cm represents the horizontal pressure, and Dm the vertical pressure. But since the angle DCn = 60°, the angle DCm = 30°, and the triangle DCm is half an equilateral triangle; $\therefore$ Dm = $\frac{1}{2}$ DC, or Dm = 10,

$$\begin{aligned} \text{and } (Cm)^2 &= (DC)^2 - (Dm)^2 \\ &= 400 - 100, \\ Cm &= \sqrt{300} = 17{\cdot}32, \end{aligned}$$

$\therefore$ Vertical pressure = 10 lbs.
Horizontal pressure = 17·32 lbs.

EXAMPLES.

1. If a weight of 40 lbs. slide along a perfectly smooth rod, inclined at an angle of 60° with the vertical line, what force is necessary to keep the weight at rest in any part of the rod? Ans. 34·64.

2. In the previous question, if the rod were inclined at an angle of 45° shew that the force necessary to keep the weight at rest will be 28·284 lbs.

3. A smooth ring sustaining a weight of 50 lbs.

slides along a cord fastened at two points, lying in the same horizontal line, find the tension in the cord, when the two parts of the cord are at right angles to each other. Ans. 35·35.

4. In the previous question, what will be the tension in the cord when the two parts of the cord form an angle of 60°. Ans. 28·9 nearly.

5. One extremity of two cords, whose lengths are severally 14 and 48 inches, is fastened at points in the same horizontal line; the other extremity is fastened to a weight of 100 lbs. and the cords are at right angles, find the tensions in the cords.

Ans. 96 in the shorter cord, and 28 in the longer.

6. Find the horizontal and vertical pressures, when a force of 100 lbs. acts in a direction making an angle of 60° with the vertical line. Ans. 86·6 and 50.

7. Find the horizontal and vertical pressures, when a force of 60 lbs. acts in a direction making an angle of 45° with the vertical line.

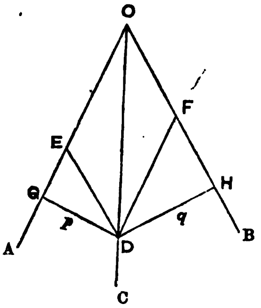
Ans. each equals 42·426 lbs.

8. Find the horizontal and vertical pressures, when a force of 80 lbs. acts in a direction making an angle of 30° with the vertical line. Ans. 40 and 69·28.

19. Def. The product of a force, and the perpendicular distance of its direction from any given point, is termed the *moment* of the force about that point. For instance, if a force exert a pressure of 20 lbs. and the perpendicular drawn to its direction from any point be 6 inches, the moment of the force about that point is 20 × 6, or 120. The moment of a force about a point serves, as will presently be seen, to measure the effect of the force in turning the body round that point.

20. PRINCIPLE V. *The moments of two concurrent forces about any point in the direction of their resultant are equal: and conversely, if the moments of two concurrent forces about any point in their plane be equal, that point will lie in the direction of their resultant.*

Let P and Q be two forces acting respectively in the lines AO and BO, and let CO be the direction of their resultant. In CO take any point D, and from D draw the perpendiculars DG and DH. Let DG $= p$ and DH $= q$.



Then shall Pp = Qq.

For draw DE parallel to OB, and DF parallel to OA. Then, since the resultant of P and Q passes along the diagonal OD, the sides of the parallelogram OEDF will represent the forces. Therefore OE = P, and OF = Q. The triangles OED, OFD are equal. But the triangle OED $= \frac{1}{2}$OE $\times p$,* and the triangle OFD $= \frac{1}{2}$OF $\times q$.

$$\therefore \quad \tfrac{1}{2}\text{OE} \times p = \tfrac{1}{2}\text{OF} \times q$$
$$\therefore \quad \text{OE} \times p = \text{OF} \times q$$
$$\therefore \quad \text{P}p = \text{Q}q.$$

Conversely. If Pp = Qq the resultant of P and Q passes through D.

$$\text{For} \quad \text{OE} \times p = \text{OF} \times q$$
$$\text{And} \quad \text{P}p = \text{Q}q$$
$$\therefore \quad \frac{\text{OE}}{\text{P}} = \frac{\text{OF}}{\text{Q}}$$
$$\text{OE} : \text{OF} :: \text{P} : \text{Q},$$

---

* The area of any triangle equals half the product of the base and height.

that is, if OE represent P on any scale, OF will represent Q on the same scale; and, consequently, by the parallelogram of forces, their resultant will pass along the diagonal OD.

21. If a body turn about a fixed point or pivot, it is clear that no force, whose direction passes through the fixed point, will cause any motion. Consequently, if two forces act upon such a body, and their resultant pass through the fixed point, no motion will be caused. But, by Principle V. the resultant will pass through the fixed point if the forces are concurrent, and their moments about that point are equal. Hence, this therefore is the only condition of equilibrium in such a case.

Ex. 1. Two concurrent forces, P and Q, act upon a body which moves about a fixed point, at distances 8 and 10 inches respectively, what must Q be if P is 50 lbs. when there is equilibrium

The only condition is, that

$$Q \times 10 = 50 \times 8,$$
$$10\,Q = 400,$$
$$\therefore \quad Q = 40.$$

Ex. 2. A force of 80 lbs. acts at a distance of 12 inches from a fixed point, at what distance must a force of 60 lbs. act, in order that there may be equilibrium?

Let x = distance required, then

$$60x = 80 \times 12 = 960,$$
$$\therefore \quad x = 16.$$

EXAMPLES.

1. Forces of 18 and 26 lbs. respectively, acting at right angles to the arms of a bent bar moving about

the vertex are in equilibrium, the shorter arm is 9 in. what is the length of the longer arm? Ans. 13 in.

2. Forces of 63 and 35 lbs. acting at the extremities of a straight bar, at angles of 45° with the bar are in equilibrium, the bar moves about a pivot placed 5 in. from one extremity, what is the length of the bar?

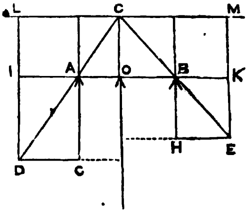
Ans. 12 in.

3. A straight bar, 13 inches in lèngth, moves about a point placed 3 inches from one extremity, a force of 30 lbs. acts at the end of the longer arm, but inclined to it an angle of 45°, what force acting at the shorter arm, at an angle of 30°, will preserve equilibrium?

Ans. 141·42 lbs.

22. PRINCIPLE VI. *The resultant of two concurrent parallel forces is concurrent with the components and equal to their sum; the resultant of two non-concurrent parallel forces is equal to their difference and concurrent with the greater; and, in all cases, the moments of the components about a point in their resultant are equal.*

Let P and Q be any two concurrent parallel forces represented by the lines GA and HB. Draw any line AB at right angles to their directions. At A and B introduce two equal and opposite forces (Principle II.) represented by IA and KB. The resultant of IA and GA is DA, and the resultant of KB and HB is EB. Remove the points of application of these two resultants from A and B to C. Resolve these into their original components, and we shall have acting along LM two equal and opposite forces, corresponding to IA and KB,



which, being in equilibrium, may be removed, and two forces equal respectively to P and Q acting along CO. The resultant, therefore, of P and Q is a force of the magnitude P + Q acting along CO.

The parallelograms GO and AL are equal, being complements of the parallelograms about the diagonal DC. Similarly the parallelograms HO and BM are equal. But the parallelograms AL and BM are equal, being upon equal bases and between the same parallels; therefore, the parallelograms GO and HO are equal. But GO is a rectangle, and therefore equals GA × AO, and similarly HO = HB × BO. Hence,

$$GA \times AO = HB \times BO$$

$$\text{that is} \quad P \times AO = Q \times BO,$$

or the moments of the components about a point O in the resultant are equal.

In the preceding, let R stand for the resultant of P and Q. Then, if R act at O, and if P and Q act at A and B in a contrary direction to that represented in the figure, P, Q, and R will be in equilibrium. But when three forces are in equilibrium, any one is equal and opposite to the resultant of the other two. Consequently, GA, a force concurrent with R, will represent the resultant of the two non-concurrent forces R and Q. But P equals GA, therefore the resultant of R and Q equals P; but R equals P + Q, therefore P equals R — Q, or the resultant of the two non-concurrent forces R and Q is equal to their difference. As already shewn, P × AO = Q × BO add to both Q × AO, then

$$P \times AO + Q \times AO = Q \times BO + Q \times AO$$

$$(P + Q) AO = Q (BO + AO)$$

$$R \times AO = Q \times AB,$$

or the moments of the components about a point A in their resultant are equal.

It will be seen, that when the component forces are concurrent, the resultant passes between them; but that when they are non-concurrent, the resultant passes outside the greater force.

23. *Two concurrent parallel forces,* P *and* Q, *act at a distance* a *from each other, to find the distance of the resultant from either force.*

Let x be the distance of the resultant from P; then, since the resultant passes between P and Q, its distance from Q will be $a - x$. Therefore

$$Px = Q(a - x) = Qa - Qx$$
$$(P + Q)x = Qa$$
$$x = \frac{Qa}{P + Q} = \frac{Qa}{R}$$

or the distance of the resultant from one of the components equals the distance between the components multiplied by the other component and divided by the resultant.

24. *Two non-concurrent forces,* P *and* Q *act at distance* a *from each other, to find the distance of the resultant from either force.*

Let P be the greater force and R their resultant, then R equals P — Q, and passes outside P. Let x be the distance of P from the resultant, then $a + x$ is the distance of Q. Therefore

$$Px = Q\,(a + x) = Qa + Qx$$
$$(P - Q)\,x = Qa$$
$$x = \frac{Qa}{P - Q} = \frac{Qa}{R}$$

or, as before, the distance of resultant from either component equals the distance between the components multiplied by the other component and divided by the resultant.

25. *To resolve a given force* P *into two parallel forces acting at given distances on each side of it.*

Let a and b be the given distances, and let X and Y be the forces required, then P will be the resultant of X and Y, consequently $X + Y = P$ and $aX = bY$. $\therefore$

$$bX + bY = Pb,$$
$$\text{or } bX + aX = Pb,$$
$$\therefore \quad X = P\frac{b}{a+b} \text{ and } Y = P\frac{a}{a+b}.$$

Hence, to find either component, multiply the given force by the distance of the other component, and divide by the distance between the two components.

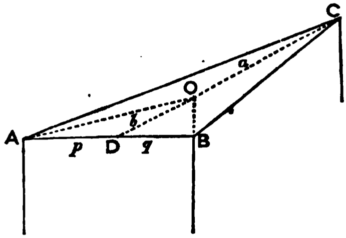
26. *To resolve a given force* P *into two parallel forces acting at given distances on the same side of it.*

Let a and b be the given distances, a being the greater, and X and Y the required forces. From art. 22 it appears that X and Y will be non-concurrent, and that the force nearer to P, namely Y, will act concurrently with P. Also, $Y - X$ equals P, and aX equals bY.

$$\therefore \quad bY - bX = Pb$$
$$aX - bX = Pb$$
$$X = P\frac{b}{a-b}, \text{ and } \therefore \ Y = P\frac{a}{a-b};$$

or, as before, to find either component, multiply the given force by the distance of the other component, and divide by the distance between the two components.

Ex. A weight of 1000 lbs. is placed upon the triangular table ABC at O, to find the pressures upon the three legs when a is 30 inches, b 10 inches, p 13 inches, and q 12 inches.



Resolving 1000 lbs. at O into pressures at C and D, we obtain—

$$\text{pressure at C} = \frac{1000 \times 10}{40} = 250,$$

$$\text{pressure at D} = \frac{1000 \times 30}{40} = 750.$$

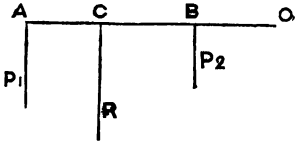
Similarly resolving the pressure at D into pressures at A and B—

$$\text{pressure at B} = \frac{750 \times 13}{25} = 390,$$

$$\text{pressure at A} = \frac{750 \times 12}{25} = 360.$$

27. *The sum of the moments of any two concurrent parallel forces, about any point in their plane lying without them, is equal to the moment of the resultant about that point.*

Let P_1 and P_2 be any two concurrent parallel forces, and let R be their resultant; let p_1, p_2 and r be their distances respectively from the point O, then shall



$$P_1 p_1 + P_2 p_2 = Rr.$$

For, since R is the resultant of P_1 and P_2, R equals $P_1 + P_2$, and $P_1 \times AC$ equals $P_2 \times BC$. But $AC = p_1 - r$ and $BC = r - p_2$, therefore

$$P_1 (p_1 - r) = P_2 (r - p_2)$$

$$P_1 p_1 - P_1 r = P_2 r - P_2 p_2$$

$$\therefore \; P_1 p_1 + P_2 p_2 = (P_1 + P_2) r = Rr.$$

EXAMPLES.

1. Find the position of the resultant of two concurrent parallel forces whose magnitudes are 245 and 125 lbs. respectively, acting at points 74 inches apart.

Ans. 25 in. from the larger force.

2. Two non-concurrent parallel forces, whose magnitudes are 340 and 120 lbs. respectively, act at points 44 inches apart; find the position of the resultant.

Ans. 24 in. from the larger force.

3. The larger of two concurrent parallel forces is a pressure of 28 lbs. the resultant is equal to 40 lbs. acting at a distance of 18 inches from the larger component; what is the distance between the components?

Ans. 60 in.

4. The larger of two non-concurrent parallel forces is 70, the resultant is 49, acting at a distance 15 from the larger component; what is the distance between the components? Ans. 35.

5. The smaller of two non-concurrent parallel forces is 15, the resultant is 45, acting at a distance 56 from the smaller component; what is the magnitude of the larger component? Ans. 45.

6. Resolve a force of 700 lbs. into two concurrent parallel forces acting at distances from it of 17 and 3.

Ans. 595 and 105 lbs.

7. Resolve a force of 1,000 lbs. into two non-concurrent parallel forces acting at distances from it of 4 and 24. Ans. 1,200 and 200.

8. A weight of 648 lbs. is placed upon the triangular table ABC (fig. art. 26), at O; what are the pressures on the legs, when $a = 27$ inches, $b = 9, p = 10$, and $q = 17$?

Ans. pressure on A = 306; pressure on B = 180; pressure on C = 162.

9. A beam AB, 10 feet in length, rests horizontally upon two vertical props, A and B, and another beam, CD, 20 feet in length, rests upon two vertical props, C and D; a third beam, 30 feet in length, lies across the two former beams in such a way that the one extremity

E is 3 feet from the prop A, and the other extremity F is 5 feet from the prop D; a weight of 60 lbs. is attached to the third beam at a distance of 10 feet from F: shew that the pressures upon A, B, C, D, are 14, 6, 10, 30 lbs. respectively.

10. A weight resting upon a horizontal table exerts an equal pressure upon each of the three legs; determine the position of the weight.

CHAPTER II.

ON THE CENTRE OF GRAVITY.

28. If any number of parallel forces act at given points in a rigid body, it can be shewn that there is a point through which the resultant will always pass, whatever the position in which the body may be placed.

This point is termed the centre of the parallel forces.

29. *Gravity* is the force which attracts every particle of matter towards the centre of the earth. The *weight* of a body is the total force with which that body is drawn towards the earth's centre: it is therefore the same as the resultant of the forces which, in consequence of the existence of gravity, act upon its several particles. These forces may, without sensible error, be regarded as parallel forces. The centre of these parallel forces is called the *centre of gravity*.

The centre of gravity of a body is, then, the point through which, in every position of the body, will pass the resultant of all the forces which, in consequence of gravity, act upon its particles.

30. It follows from the definition, that if the centre of gravity of a body be fixed, the body will rest in any position. For in every position the resultant of all the forces arising from gravity passes through the fixed point, and being met by the resistance of that point, equilibrium is preserved.

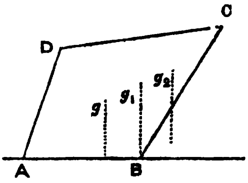
31. Since the resultant may be always substituted in place of its component forces, it follows that in considering the influence of any weighty body in producing equilibrium, we may substitute a force equal to the weight of the body, and acting at its centre of gravity.

32. *If a body suspended from any point be at rest, the centre of gravity must lie in the vertical line drawn through this point.* For there are two forces in equilibrium; viz., the weight of the body which acts vertically, and at the centre of gravity, and the reaction of the fixed point. By Principle I. these forces must lie in the same straight line, and therefore the vertical line through the centre of gravity must pass through the fixed point.

Hence, if a body be suspended successively from two different points, and the vertical lines passing through those points be drawn, the intersection of these two lines will be the centre of gravity.

33. *A body placed with its base upon a plane surface will stand or fall, according as the vertical line through its centre of gravity falls within or without the base.*

Let ABCD be any body, whose base AB rests upon a plane surface. Let the centre of gravity be at g, so that the vertical line through g falls between AB. The weight of the body acting in this line is met by the resistance of the plane, and, consequently, will not cause the body to turn about either A or B.



But if the centre of gravity be at g_2, so that the vertical line drawn through the centre of gravity falls without the base, the weight of the body, acting in this line, is not met by the re-action of the plane. There cannot, therefore, be equilibrium, but the body will turn over on to the side BC.

If the centre of gravity be at g_1, so that the vertical line drawn through it pass through one of the extremities of the base, the body will rest; for the weight acting in the vertical line is met by the resistance of the plane; a very slight disturbance will, however, cause the body to fall over.

34. The centre of any perfectly symmetrical figure must also be the centre of gravity, for there can be no reason why the centre of gravity should fall upon one side of that point more than upon another. Hence, the centre of gravity of a line is its point of bisection; of a parallelogram, the intersection of its diagonals; of a parallelòpiped, the intersection of its diagonals; of a circle and sphere, their centres.

35. *To find the centre of gravity of a triangle.* Let ABC be the given triangle. Bisect AB in D and AC in E. Draw CD, BE. If we suppose the triangle to be made up of an infinite number of lines parallel to AB, these lines will be bisected by CD. Consequently, the centre of gravity of each of these lines will be in the line CD, and therefore the centre of gravity of the triangle will lie in the line CD. For similar reasons the centre of gravity must lie in the line BE. Therefore g, the intersection of these two lines, must be the centre of gravity required.

Join the points DE. DE will be parallel to BC* and equal to half BC. Hence, the triangle gED is equiangular with the triangle gBC, and the sides of the former are each half of the corresponding sides of the latter. Therefore $g\text{D} = \frac{1}{2}\, g\text{C} = \frac{1}{3}\,\text{CD}$.

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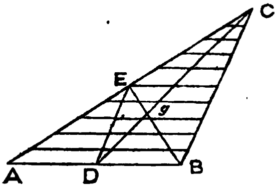
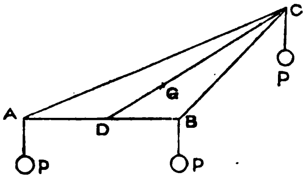
* The line joining the bisections of the sides of a triangle is parallel to the base and equal to half the base. Hence it may



Hence, the centre of gravity of a triangle lies in the line joining any vertex with the bisection of the opposite side, at a distance from the side of one-third of this line.

36. *To find the centre of gravity of three equal weights suspended from the angular points of a triangular plate; the plate being supposed to be without weight.*

Let a weight P be suspended from each of the angular points A,B,C of the triangular plate ABC. Since these weights hang vertically, they may be regarded as parallel forces acting severally at the points of suspension. Bisect AB in D. Join DC. Let DG $= \frac{1}{3}$ DC $= \frac{1}{2}$ GC. The resultant of the force P at A, and the force P at B, will be a force 2 P acting



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easily be proved, that if two triangles be equiangular, and the base of one be half the base of the other, the sides of the one are each half of the corresponding sides of the other. For if the sides of the larger triangle be bisected, and the bisections joined, a triangle will be formed equal in every respect to the smaller of the given triangles.

at D. The resultant of 2 P at D and P at C, will pass through G; for since DG $= \frac{1}{2}$ GC,

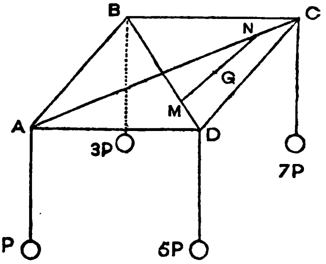
$$2\,P \times DG = P \times GC,$$

G is therefore the centre of gravity required.

By the preceding section, G is the centre of gravity of the triangle ABC, whence it appears that the centre of gravity of a triangle corresponds with that of three equal forces acting at its angular points.

37. *To find the centre of gravity of four weights,* P, 3P, 7P, *and* 5P, *suspended from the angular points of a square plate.*

Let ABCD be the square plate, and let the weights P, 3P, 7P, and 5P be suspended severally from the points A, B, C, D. Draw the diagonals, AC, BD. Take N so that CN $= \frac{1}{8}$ A C, and M so that DM $= \frac{3}{8}$ BD. Then AN $= \frac{7}{8}$ AC, and BM $= \frac{5}{8}$ BD.

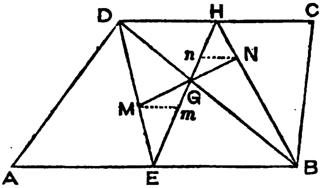


The resultant of P at A, and 7P at C is a force 8P at N, since 7P $\times$ CN $=$ P $\times$ AN. And the resultant of 3P at B and 5P at D is also a force 8P at M; since 5P $\times$ DM $=$ 3P $\times$ BM. We have then in the place of the original forces two equal forces acting at M and N. Bisect MN at G, then the resultant of these equal forces will pass through G, and G is the centre of gravity required.

38. *To find geometrically the centre of gravity of a trapezium.*

Let ABCD be the trapezium, in which the sides AB, CD are parallel.

Bisect AB in E, and DC in H, and join HE. If the trapezium be supposed to be made up of an infinite number of lines parallel to AB, each of these lines will be bisected by EH, and consequently the centre of gravity of the trapezium must be in the line EH.



Join DE, and take EM = $\frac{1}{3}$ ED. Similarly join HB, and take HN = $\frac{1}{3}$ HB. Then M is the centre of gravity of the triangle ABD, and N that of the triangle BCD. Consequently the centre of gravity of the whole figure must be in the line MN; but it also lies in the line EH, therefore the intersection of these lines or the point G is the centre of gravity required.

EXAMPLES.

1. Find the centre of gravity of three weights, 5P, 4P, and 6P, suspended from the angular points of a triangular plate, the plate being supposed to be without weight.

2. If a triangular plate ABC form an isosceles triangle, whose base is AB, and whose height is 12 inches, find the distance of the centre of gravity from C, when weights 10, 10, and 40 lbs. are suspended from A, B, and C respectively; the plate being supposed to be without weight. Ans. 4 inches.

3. In the preceding example, if the plate weigh 20 lbs. what is the distance of the centre of gravity from C? Ans. 5 inches.

4. In example 2, if ABC be an equilateral triangle, whose side is 20 inches, find the distance of the centre of gravity from C. Ans. 5·77.

5. Find geometrically the centre of gravity of half a regular hexagon.

6. Find geometrically the centre of gravity of half a regular octagon.

7. Find geometrically the centre of gravity of the quadrilateral figure, formed by two isosceles triangles, standing upon a common base.

CHAPTER III.

ON THE MECHANICAL POWERS.

39. A *machine* is an instrument, by the agency of which one force can resist or overcome another force not immediately opposite to it in direction.

40. The simplest of such instruments are denominated the *Mechanical Powers*. These are, the lever, the wheel and axle, toothed wheels, the pulley, the inclined plane, the wedge, and the screw. These differ from each other more in their structure than in the principles of their operation, for when in equilibrium, the wheel and axle, the toothed wheel, and the pulley may be reduced to the lever; and the wedge and the screw are but modifications of the inclined plane.

The two forces which act upon either of these simple machines are, for the sake of distinction, called the *power* and the *weight*, the latter always denoting the force to be resisted or overcome.

41. By the *mechanical advantage* of any machine is meant the ratio of the weight to the power, when in equilibrium; thus, if a power of 2 lbs. sustain a weight of 30 lbs. the mechanical advantage is $30 \div 2$ or 15.

42. THE LEVER.—The Lever is an inflexible bar, capable of free motion about a fixed axis, called the fulcrum. Unless the contrary be stated, the lever is usually supposed to be without weight.

Levers are of three kinds, according to the relative position of the power, weight, and fulcrum.

When the fulcrum is between the power and the weight, the lever is of the first kind.

When the weight is between the fulcrum and the power, the lever is of the second kind.

When the power is between the fulcrum and the weight, the lever is of the third kind.

The beam of a balance is a lever of the first kind, an oar is a lever of the second kind, and the treadle of a lathe is a lever of the third kind.

Scissors are double levers of the first kind, nut-crackers are double levers of the second kind, and spring shears are double levers of the third kind.

43. *To find the condition of equilibrium in the lever, when the power and the weight act in parallel directions.*

According to what has been stated in Article 21, there will be equilibrium, if the resultant of the two forces passes through the fulcrum. Let P represent the power and W the weight; let a be the distance from the fulcrum at which P acts, and b the distance at which W acts. Then since P and W are by hypothesis parallel forces, their resultant will pass through the fulcrum if

$$Pa = Wb,$$

whether the fulcrum be between the forces, as in levers of the first kind, or without them, as in levers of the second and third kinds. Hence, when the power and the weight are parallel forces, the condition of equilibrium in a lever of any kind, is that *the power* × *its distance from fulcrum = the weight* × *its distance from the fulcrum;* and, therefore, the

$$\text{mechanical advantage} = \frac{\text{dist. of power from fulcrum}}{\text{dist. of weight from fulcrum}}.$$

Ex. 1. A weight of 20 lbs. is suspended at a distance of 3 inches from the fulcrum of a lever of the first kind, where must a power of 4 lbs. be suspended, in order that there may be equilibrium?

Let x = the distance sought, then

$$4x = 20 \times 3 = 60,$$
$$x = 15 \text{ inches.}$$

Ex. 2. In a lever of the second kind, whose length is 18 inches, where must the weight be placed if P = 7, when W = 42? Let x be distance of W from fulcrum, then

$$42x = 7 \times 18 = 126,$$
$$x = 3.$$

44. *To find the condition of equilibrium in a lever acted upon by parallel forces, when the weight of the lever itself is regarded.*

Let P denote the power and W the weight. Let a = the length of the arm at which P acts, and b = the length of the arm at which W acts. Let w = the weight of the lever, and c = the distance of its centre of gravity from the fulcrum.

The weight of the lever may be regarded as a force w acting at the centre of gravity, which may act concurrently with either the power or the weight. First, let it act concurrently with P, and let R be the resultant of P and w, then there will be equilibrium if the moment of R about the fulcrum equals the moment of W about the fulcrum. But by Article 27, the moment of R about the fulcrum is $Pa + wc$; therefore, the condition of equilibrium is

$$Pa + wc = Wb.$$

Secondly, let the weight of the lever act concurrently with W, and let R be the resultant of W and w. The moment of R about the fulcrum must equal the moment of P, and hence the condition required is

$$Pa = Wb + wc.$$

Ex. The two arms of a lever, of uniform thickness and density, are 7 in. and 3 in. respectively in length; a weight of 12 lbs. is suspended from the shorter arm, what power will support it, the weight of the lever being 4 lbs?

Since the lever is of uniform thickness and density, its centre of gravity is at half its length, and therefore at a distance of 2 in. from the fulcrum. Let P be the force required, then

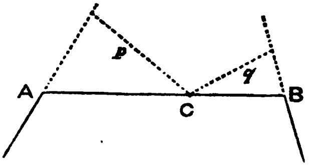
$$P \times 7 + 4 \times 2 = 12 \times 3,$$

or $$7P + 8 = 36,$$

$\therefore$ $$P = 4 \text{ lbs}.$$

45. *To determine the condition of equilibrium in a lever, when the power and weight act in any directions whatever.*

Let P and Q be forces acting in any direction at the extremities A and B of any lever, which may be either straight as in the figure, or bent in any manner. From the fulcrum C let fall perpendiculars upon the directions of the forces, and let their lengths be p and q respectively. Then (Principle V.) the resultant of P and Q passes through C, if



$$Pp = Qq,$$

which is consequently the condition of equilibrium required.

46. *To determine the pressure upon the fulcrum of a lever.*

The pressure upon the fulcrum is in every case the resultant of the forces acting upon the lever. When these are parallel and act concurrently, the resultant equals their sum;—if not concurrent, the resultant equals their difference. Hence the pressure upon the fulcrum,

in levers of the first kind, is $P + W$,
.... second .... $W - P$,
.... third .... $P - W$.

If the weight of the lever itself be regarded, it must be added to the power or the weight, according as it acts concurrently with the one or with the other. Then if $w =$ the weight of the lever, the pressure upon the fulcrum, in levers of the

first kind, is $P + W + w$,
second ...... $W - (P + w)$ or $(W + w) - P$,
third ...... $(P + w) - W$ or $P - (W + w)$.

47. An ordinary balance is, mechanically considered, simply a lever of the first kind, provided with pans suspended from its extremities. When the arms of the balance are equal, a body placed in one of the pans will be balanced by an equal weight in the other. When, however, the arms are unequal, the weight in one pan is not equal to that of the body in the other.

To determine the true weight of a body by a false balance.

Let w be the true weight of the body, and let x and y be the unknown arms of the balance. When w is placed in one pan, let it be balanced by a weight of 16 lbs.; and when placed in the other, by a weight of

12 lbs. Then, since the weights w and 16, acting at the arms x and y, were in equilibrium,

$$wx = 16y,$$

and since w and 12, acting at the arms y and x, were in equilibrium,

$$wy = 12x.$$

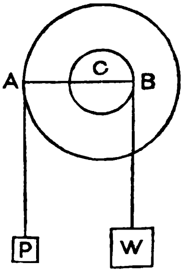
Multiplying together $w^2xy = 192\ xy$

$$w^2 = 192 \text{ or } w = \sqrt{192},$$

the true weight is a mean proportional between the two false weights.

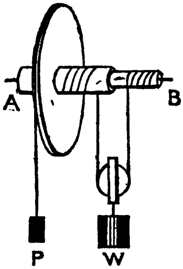
48. **The Wheel and Axle.**—The wheel and axle consists of a cylinder or axle firmly fixed to a wheel, and having a common axis with it. The weight is attached to a cord passing round the axle, and the power to a cord passing round the wheel.

Let the figure represent a section of such a machine, where C is a point in the common axis, CA the radius of the wheel, and CB the radius of the axle. Let P be the power and W the weight. These may be regarded as two parallel forces, and if in equilibrium, their resultant must pass through C, and therefore we must have P × AC = W × BC, or P × radius of wheel = W × radius of axle. Hence the mechanical advantage in the wheel and axle is found by dividing the radius of wheel by the radius of the axle.



49. The advantage in the wheel and axle may be increased either by increasing the radius of the wheel or by diminishing the radius of the axle. If, however,

the wheel be very greatly increased, the machine becomes too unwieldy to be serviceable, and if the axle be much diminished, it becomes too weak to sustain the weight. These difficulties in the way of an indefinite increase of the mechanical advantage are overcome by the simple device of a compound axle, one-half of which is of smaller radius than the other. One end of the cord, sustaining the weight, is wound round the thicker part of the axle, and the other end, in a contrary direction, round the thinner part. As the power descends, some part of the cord unwinds from the thinner axle, while another part is wound up around the thicker axle; but as more of the cord is wound up than is let out, the weight is raised by the action of the machine. Let P be the power, W the weight, r the radius of wheel, a the radius of thicker axle, and b the radius of the thinner axle. Since the whole weight is supported by the two parts of the cord, the tension in the cord $= \frac{1}{2}W$.



The power and the tension in the cord passing to the thinner axle both act on the one side of the axle, and the tension in the cord passing to the thicker axle acts on the other. There will be equilibrium if the moment of the resultant of the former, about a point in the axis, equals the moment of the latter, and therefore, by Article 27, if the sum of the moments of the former equals the moment of the latter;

$$\therefore \quad Pr + \tfrac{1}{2}Wb = \tfrac{1}{2}Wa,$$

$$\text{or} \quad Pr = \tfrac{1}{2}W(a - b)$$

$$\therefore \quad P : W :: \tfrac{1}{2}(a - b) : r.$$

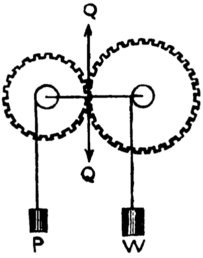
Whence it appears that the mechanical advantage is equal to twice the radius of the wheel divided by the difference of the radii of the axles. Part of this advantage is owing to the introduction of the pulley, which it will be presently seen doubles the advantage of the machine. Consequently, the advantage of the wheel and axle alone is equal to the radius of the wheel divided by the difference of the radii of the axles, or the machine is equivalent to a simple wheel and axle, having an axle equal to the difference between the thicker and thinner parts of the compound axle.

50. TOOTHED WHEELS.—A toothed wheel is a circular plate of wood or metal, having its circumference indented or cut into equal teeth all the way round. If two such wheels, having their teeth of the same magnitude, and at the same distance apart, be so placed that a tooth of one may lie between any two of the other, then if one of them be turned round by any means the other will be turned round also.

If the teeth be large, the amount and direction of the pressure which the one wheel exerts upon the other will, unless the teeth be made of a peculiar shape, vary considerably as the wheels revolve; but if the teeth be small in comparison with the size of the wheel, this variation may be disregarded, and the mutual pressure may without any great error be treated as constant in magnitude and direction.

51. *To find the ratio of the power and weight in toothed wheels, when the teeth are small.*

Let the power and the weight both act at axles of equal radius, and let c be this radius. Let a be the radius of P's wheel, and b the radius of W's wheel.



Since the teeth are small, the pressure of the one wheel upon the other may be regarded as constant, and as acting in the direction of the common tangent to the two wheels. Let Q denote this pressure. Then since P and Q are in equilibrium,

$$Pc = Qa,$$

Similarly $$Wc = Qb,$$

$\therefore$ multiplying crosswise $$PQbc = WQac,$$

$$\therefore \quad Pb = Wa,$$

$$\text{or} \quad P : W :: a : b.$$

But since the teeth in each wheel are of the same magnitude and at the same distance apart, the number of teeth in each wheel will be proportional to the circumference, and consequently to the radius. Therefore, P : W :: number of teeth in P's wheel : number of teeth in W's wheel.

52. THE PULLEY.—A pulley consists of a small wheel, which moves freely about an axis, and allows a cord to pass over any part of its circumference. Unless it be otherwise stated, the wheel is supposed to revolve without friction, and the cord to be perfectly flexible.

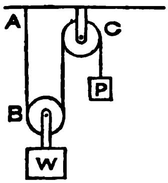
No mechanical advantage is gained by a fixed pulley; for, as the tension in every part of the cord is the same, if a weight W be suspended at one extremity, an equal weight must be applied at the other to maintain equilibrium. Hence in this case

$$P = W.$$

The effect of a fixed pulley is simply to change the direction of a force.

53. *To find the ratio of the power and weight in a single moveable pulley when the cords are parallel.*

Let the weight W be attached to the moveable pulley B, and let B be sustained by a cord ABC, one extremity of which is fastened at A, and the other after passing over the fixed pulley C sustains the power P.



Let there be equilibrium:—then the weight W is sustained by the tension in BA and the tension in BC; but since the cords are parallel, these tensions may be regarded as two parallel forces, and therefore W must equal their sum. But the tension of the cord is the same throughout, and is equal to P,

$$\therefore \quad W = 2P \quad \text{or} \quad P = \tfrac{1}{2} W.$$

54. The following combinations of pulleys are termed respectively, the first, second and third system of pulleys.

In the first system of pulleys, each pulley hangs by a separate cord, one end of which is fastened to a fixed beam, and the other to the pulley above it. (See fig. Art. 55.)

In the second system of pulleys, the same cord passes round all the pulleys, which are arranged in two blocks, one of which is fixed and the other bears the weight. (See fig. Art. 57.)

In the third system of pulleys, each cord is attached to the weight. (See fig. Art. 58.)

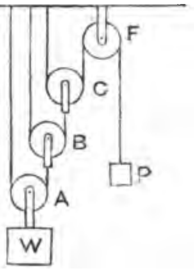
55. *To find the ratio of the power and weight in the first system of pulleys, when the weight of the pulleys is disregarded.*

Let ABC be three moveable pulleys, and let P and W be in equilibrium.

By Article 53,

tension in AB $= \frac{1}{2}W$,
tension in BC $= \frac{1}{2}$ tension in AB $= \frac{1}{4}W$,
tension in CF $= \frac{1}{2}$ tension in BC $= \frac{1}{8}W$.

But, by Article 52, tension in CF $=$ P, therefore, when there are three moveable pulleys, $P = \frac{1}{8}W$ or $W = 8P$. Similarly, if there be four pulleys, $W = 16P$; if five, $W = 32P$, and so on. Hence, in the first system of pulleys, when the weight of the pulleys is disregarded, the weight is found by doubling the power as many times as there are pulleys, and the power is found by halving the weight as many times as there are pulleys.



56. *To find the ratio of the power and weight in the first system of pulleys when the weight of the pulleys is regarded.*

The total weight acting at each pulley is the tension of the string attached to the block, together with the weight of the block.

The entire weight at A is the weight W, together with the weight of the pulley A; one-half of this sum will give the tension in AB.

The entire weight at B is the tension in AB, together with the weight of the pulley B; one-half of this sum will give the tension in BC.

The entire weight at C is the tension in BC, together with the weight of the pulley C; one-half of this sum will give the tension in CF, which is equal to the power P.

Hence, when W is given to find P, to W add the weight of the lowest pulley, and divide by 2; add the weight of the next pulley, and again divide by 2. Repeat this process as many times as there are moveable pulleys, the result will give P.

If P be given, W is found by the inverse processes. Double P and subtract the weight of the highest pulley. Double again, and subtract the weight of the next pulley, and so on as many times as there are moveable pulleys.

Ex. 1. Three pulleys are arranged according to the first system, the lowest weighs 6 lbs., the next 5, and the highest 4, what power will sustain a weight of 1000 lbs.?

$$
\begin{array}{l}
1000 \\
6 \\
\overline{1006} \div 2 = 503 \\
5 \\
\overline{508} \div 2 = 254 \\
4 \\
\overline{258} \div 2 = 129 = P.
\end{array}
$$

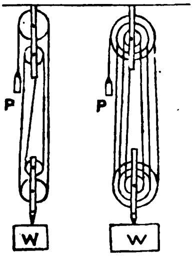
Ex. 2. Four pulleys are arranged according to the first system, the lowest weighs 4 lbs., the next 5, the next 3, and the highest 6 lbs., what weight will be sustained by a power of 43 lbs.?

$$
\begin{array}{l}
43 \times 2 = 86 \\
6 \\
\overline{80} \times 2 = 160 \\
3 \\
\overline{157} \times 2 = 314 \\
5 \\
\overline{309} \times 2 = 618 \\
4 \\
\overline{614} = W.
\end{array}
$$

57. *To find the ratio of the power and weight in the second system of pulleys.*

Since there is but one cord, and P is attached to one extremity of it, the tension in every part is equal to P. Hence if n be the number of the portions of this cord in contact with the lower block, the weight supported will be nP; therefore in this system,

$$W = nP \text{ or } P = \frac{1}{n} W.$$



If the weight of the lower block be w, the total weight supported is W + w; therefore, if the weight of the block be regarded,

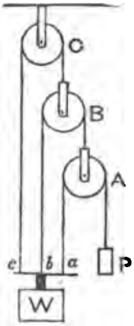
$$W + w = nP, \text{ or } P = \frac{1}{n}(W + w).$$

58. *To find the ratio of the power and weight in the third system of pulleys, when the weight of the pulleys is disregarded.*

Let there be three pulleys ABC; then tension in Aa = P,
tension in Bb = pressure on A = 2P,
tension in Cc = pressure on B = 4P.

But as the weight is supported by the tensions in the three cords Aa, *Bb*, and Cc, and since the cords are parallel, W must equal the sum of the tensions; therefore

$$W = P + 2P + 4P = 7P.$$



Similarly, if there be four pulleys,

$$W = P + 2P + 4P + 8P = 15P.$$

Hence, if as many terms of series 1, 2, 4, 8, &c. as there are pulleys be added together, the sum multiplied by the power will give the weight; and consequently, the weight divided by this sum will give the power.

Ex. 1. Five pulleys are arranged according to the third system, what weight will be supported by a power of 12 lbs., the weight of the pulleys being disregarded?

$$1 + 2 + 4 + 8 + 16 = 31,$$
$$W = 31 \times 12 = 372 \text{ lbs.}$$

Ex. 2. Six pulleys are arranged according to the third system, what power will support a weight of 504 lbs., the weight of the pulleys being disregarded?

$$1 + 2 + 4 + 8 + 16 + 32 = 63,$$
$$\therefore \quad P = 504 \div 63 = 8 \text{ lbs.}$$

59. *To find the ratio of the power and weight in the third system of pulleys, when the weight of the pulleys is regarded.*

The tension in A*a* (fig. Art. 58) equals the power. The tension in B*b* equals twice the power increased by the weight of the lowest pulley. The tension in C*c* equals twice the tension in B*b*, increased by the weight of the second pulley. The weight W will be equal to the sum of the three tensions.

Hence generally, add together as many terms of the following series as there are pulleys—viz., the power, twice the power + weight of the lowest pulley, twice the preceding + the weight of the next pulley, and so on; this sum will be equal to the weight supported.

Ex. 1. Four pulleys, weighing respectively 6, 5, 4, and 3 lbs. are arranged according to the third system, the last mentioned being the lowest, what weight will be sustained by a power of 16 lbs?

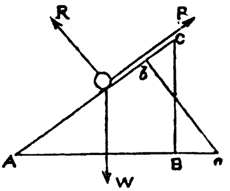
$$\begin{array}{rrcr} & & \text{P} = & 16 \\ 2 \times & 16 + 3 & = & 51 \\ 2 \times & 51 + 4 & = & 106 \\ 2 \times & 106 + 5 & = & \underline{217} \\ & & \text{W} = & 390 \text{ lbs.} \end{array}$$

Ex. 2. With the same set of pulleys, find the power which will support a weight of 728 lbs.

$$\begin{array}{lcl} \text{tension in first string} & = & \text{P} \\ \quad\text{,,} \quad \text{second ,,} & = & 2\text{P} + 3 \\ \quad\text{,,} \quad \text{third ,,} & = & 4\text{P} + 10 \\ \quad\text{,,} \quad \text{fourth ,,} & = & \underline{8\text{P} + 25} \\ \therefore \qquad \text{W} & = & 15\text{P} + 38 = 728 \\ & & \qquad 15\text{P} = 690 \\ & & \qquad\quad \text{P} = 46 \text{ lbs.} \end{array}$$

60. THE INCLINED PLANE.—*To find the ratio of the power and weight in the inclined plane, when the power acts parallel to the plane.*

Let W be a weight resting upon the inclined plane ABC, and supported by a power P acting parallel to the plane. Make A*b* = AB and A*c* = AC, then the triangle A*bc* is in all respects equal to the triangle ABC. In the triangle A*bc*,



bc is perpendicular to the direction of the power P.

A*c* ,, ,, the direction of gravity.

A*b* ,, ,, the re-action of the plane.

∴ by Principle IV. P : W :: *bc* : A*c*,

or P : W :: BC : AC.

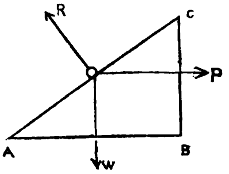
That is, P is to W as the height of the plane is to its length. In a similar manner the pressure of W on the plane may be determined. This is equal and opposite to the reaction of the plane. Let R denote this re-action,

$$R : W :: Ab : Ac,$$
$$:: AB : AC.$$

That is, the pressure on the plane is to W as the base of the plane is to its length.

61. *To find the ratio of the power and weight in the inclined plane, when the power acts horizontally.*

In this case W is kept at rest by three forces, viz., the force of gravity acting vertically, the power acting horizontally, and the re-action of the plane acting perpendicularly to the plane. The sides of the triangles ABC are severally perpendicular to the directions of these forces, viz.—



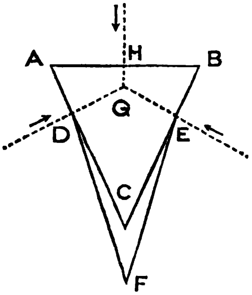
BC is perpendicular to the direction of P,
AB .... W,
AC .... R.

Therefore P : W : : BC : AB, or the power is to the weight as the height of the plane is to the base.

Also, R : W : : AC : AB, or the pressure on the plane is to the weight as the length of the plane is to the base.

62. THE WEDGE.—*To find the ratio of the power and the resistance in an isosceles wedge.*

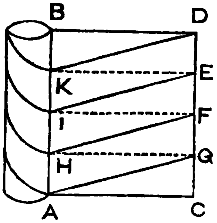
Let ABC be the section of an isosceles wedge introduced into the cleft DFE, and let the points DE be similarly situated on the two sides of the wedge. The resistance on each side of the wedge will be the same, and if R = the total resistance, the resistances at D and E will each = $\frac{1}{2}$R; and they act perpendicularly to the sides of the wedge. Let a power P act at the point H, the centre of the back of the wedge. The directions of these three forces when produced will meet in a point G, they may therefore be considered as three forces acting upon a point, and in equilibrium. The sides of the triangle ABC are severally perpendicular to the directions of the three forces, and therefore,



$$P : \tfrac{1}{2}R :: AB : AC,$$
$$\therefore \quad P : R :: \tfrac{1}{2}AB : AC.$$

That is, the power is to the total resistance, as half the back of the wedge is to the side of the wedge.

63. The Screw.—Let AB be a cylinder, and AD a rectangle, whose base AC is equal to the circumference of the cylinder. Let the side CD be divided into any number of equal parts. Join AG, and through the points F, E, D, draw FH, EI, DK parallel respectively to AG. Then if the rectangle AD be wrapped round the cylinder, the parallel



lines AG, HF, &c. will trace out the continued spiral line called the screw.

Hence any resistance to be overcome by a screw, may be regarded as a weight resting upon an inclined plane, whose base is equal to the circumference of the cylinder, and whose height is equal to the distance between the threads of the screw.

When the screw is used for mechanical purposes, the power is always applied in a direction perpendicular to the axis of the cylinder, and consequently parallel to the base of the inclined plane forming the screw line; and therefore

P : W :: height of inclined plane : the base;

that is, P : W :: distance between the threads of the screw : circumference of the cylinder.

64. *To find the mechanical advantage of any combination of machines.*

In a combination of machines, the weight of the first machine is the power of the second, the weight of the second the power of the third, and so on. Let a, b, c be the separate advantages of three machines in combination. Let P and Q be the power and weight in the first, Q and R those in the second, R and W those in the third, then

$$Q = aP$$

$$R = bQ$$

$$W = cR$$

Multiplying together $QRW = abc\,PQR$

$$W = abc\,P.$$

Therefore abc represents the mechanical advantage of the combination; that is, the advantage of the combination is equal to the product of the separate advantages of the component machines.

EXAMPLES.

1. A bar 46 in. long, supposed to be without weight, is used as a lever of the first kind, where must the fulcrum be placed when a power 40 sustains a weight 190.
Ans. 8 in. from the weight.

2. If the same bar be used as a lever of the second kind, where must the weight be placed?
Ans. 9·7 in. nearly from the fulcrum.

3. If the shorter arm of a lever of the first kind be 6 in. what is the length of the lever if P be 72 when W is 648? Ans. 5 feet.

4. A bar 3 feet long, and weighing 6 lbs. is used as a lever of the first kind, the shorter arm is 8 in. what is P when W is 60? Ans. 15.

5. With the same bar, what must be the length of the shorter arm if P be 13 when W is 77?
Ans. 6 in.

6. The same bar is used as a lever of the second kind, the weight of the bar acting concurrently with W, what is P if W be 120 lbs. and W's arm be 6 in.?
Ans. 23 lbs.

7. If the weight of the bar act concurrently with P, what will P be? Ans. 17 lbs.

8. In the simple wheel and axle, the radius of the axle is 2 in. what must be the radius of the wheel if P be 10 when W is 160. Ans. 2 ft. 8 in.

9. The two radii of a compound axle are 4 and 3 inches, the radius of the wheel is 30 inches, what is P when W is 720 lbs.? Ans. 12 lbs.

10. In Example 6, what is the pressure on the fulcrum? Ans. 103 lbs.

11. Four pulleys, whose weights, beginning with the highest, are 3, 4, 2, and 6 lbs. respectively, are

arranged according to the first system, what power will sustain a weight of 462 lbs.? Ans. 32 lbs.

12. In the same system of pulleys, what weight will be sustained by a power of 62 lbs.? Ans. 942 lbs.

13. If four pulleys, weighing respectively 3, 7, 4, and 9 lbs. are arranged according to the first system, which is the most, and which the least, advantageous arrangement?

14. What weight will be sustained by a power of 100 lbs. in the most advantageous arrangement, according to the first system, of four pulleys weighing respectively 4, 8, 6, and 10 lbs. Ans. 1526 lbs.

15. With the same set of pulleys, what weight will be sustained by a power of 100 lbs. in the least advantageous arrangement? Ans. 1476.

16. If the same pulleys be arranged according to the third system, what power will sustain a weight of 1554 lbs. in the most advantageous arrangement? Ans. 100 lbs.

17. How many pulleys at least must be employed in the first system in order that 1500 lbs. may be raised by 30 lbs. if each pulley weighs 4 lbs.? Ans. 6.

18. On an inclined plane, which rises 5 in 13,* what power, acting parallel to the plane, will support a weight of 91 lbs.? Ans. 35 lbs.

19. If 1000 lbs. be supported on an inclined plane, rising 7 in 25, by a power acting parallel to the plane, what is the pressure on the plane. Ans. 960 lbs.

20. What horizontal force will sustain 1000 lbs. on an inclined plane, rising 2 in 25? Ans. 80·3 nearly.

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* An inclined plane is said to rise 5 in 13, if when its height is 5 its *length* is 13.

21. What weight will be sustained by a horizontal force of 100 lbs. on an inclined plane rising 3 in 20? Ans. 659·1 lbs.

22. What is the pressure on the plane when a weight of 504 lbs. is sustained by a horizontal force on an inclined plane rising 16 in 65? Ans. 520 lbs.

23. The diameter of a screw is 2 inches, and the distance between the threads is $\frac{1}{4}$ inch, what is the mechanical advantage? Ans. 25$\frac{1}{7}$ nearly.

24. In a common screw press, the diameter of the screw is 3 inches, the distance between the threads is $\frac{1}{4}$ inch, and the length of the lever is 3 feet, what is the mechanical advantage? Ans. 905$\frac{1}{7}$ nearly.

25. Four levers whose arms are severally 10 and 2, 8 and 3, 15 and 1, 12 and 2, are used in combination, what is the mechanical advantage? Ans. 1200.

CHAPTER IV.

ON THE LAWS OF MOTION AND THE MOTION OF FALLING BODIES.

65. When forces not in equilibrium act upon a body, motion must ensue. The degree of rapidity with which the body moves is termed its velocity.

66. Velocity may be either uniform or variable. Velocity is uniform when the body describes *all* equal spaces in equal times, and may therefore be measured by the space described in a unit of time. It is customary to take a second for the unit of time, and a foot for the unit of measure. Thus, if a body moving with a uniform velocity have passed through 1000 feet in 8 seconds, its velocity is 1000 ÷ 8 or 125; and generally, if s be the space in feet through which a body has passed in t seconds, then if v denote the velocity,

$$v = \frac{s}{t} \text{ or } s = vt.$$

Velocity is variable when *all* equal spaces are *not* described in equal times. When a body in motion has a variable velocity, its velocity at any moment may be measured by the space which would be described in a unit of time, if at that moment the velocity were to cease to vary.

67. FIRST LAW OF MOTION.—*A body in motion, not acted on by any external force, will continue to move in a straight line and with a uniform velocity.*

This is equivalent to the assertion, that matter possesses no inherent power of changing the direction or the state of its motion.

The truth of this law must be decided by an appeal to experiment. The powers with which matter has been endowed can evidently be learnt only by observation. Every attempt to prove their existence by *a priori* demonstration will be found to assume in some way or other the property under consideration.

The first law of motion cannot, however, be established by any *direct* experiment, for the prescribed conditions can under no circumstances be fulfilled. No where can we find a body which is not acted on by some external force. Every particle of matter is subject to a variety of external influences. Thus, if a ball be rolled along the ground, it is acted on by the attraction of all surrounding matter, by the resistance of the atmosphere, and by the force of friction; and it moves neither in a straight line nor with a uniform velocity. It is found, however, that the more we lessen the influence of external force, the more nearly does the motion become direct and uniform.

If a ball be rolled along a smooth pavement, it will move for a longer time, and in a line more nearly straight than when thrown with the same velocity along a rough road; and still more so if rolled along a sheet of ice.

If a weight suspended by a thread from any point be made to oscillate, it will after a time come to rest. One of the external forces acting upon the body is the resistance of the air. If this be diminished by causing the body to move within the exhausted receiver of an air pump, the oscillation will continue for a longer period;—and the more perfect the vacuum the longer will the motion continue.

On a railway, after a train has acquired the desired velocity, it is no longer necessary for the engine to work, except so far as to overcome the effects of external forces, such as the friction of the rails and the resistance of the atmosphere.

The most convincing evidence of the truth of this law is found in the accordance of the consequences deducible from it with observed phenomena. It is impossible to doubt the correctness of a principle, upon the assumption of which the motions of the moon can be predicted with almost unerring certainty, and the time of an eclipse foretold within the fraction of a second.

68. It follows from the first law of motion, that if a force continue to act upon a body already in motion, its velocity will continually change.

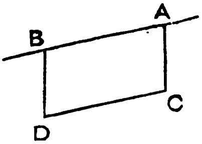
On this account force when acting upon a body in motion is termed *accelerating force;* and that, too, whether its effect be to accelerate or to retard the motion of the body.

If the change in the velocity resulting from the action of any force be uniform; that is, if in equal periods of time there be an equal increase or an equal diminution in the velocity, then the force is called a *uniform accelerating force*, and the force itself is measured by the velocity it generates (or destroys) in a unit of time. Thus, when we speak in dynamics of a force 32, we mean a force which generates in each second of time a velocity of 32 feet per second; that is, in one second it generates a velocity 32, in two seconds a velocity 64, in three seconds a velocity 96, and so on. And hence generally, if f represent any force measured, as just explained, by the velocity it generates per second, then if v be the velocity at the expiration of t seconds,

$$v = ft.$$

69. Second Law of Motion.—*When a force acts upon a body in motion, the change of motion produced is the same, both in magnitude and direction, as if the force acted on the body at rest.*

Thus, if a body move along the line AB, with such a velocity, that it would describe the space AB in one second; and if, when it arrives at A, a force act upon it such as of itself to cause a body to pass from A to C in one second, then at the end of the second the body will be found at D; the change of motion represented by BD being the same in magnitude and direction as if the force had acted upon the body when at rest. Each force produces its full effect in its own direction.



This law is proved by such experiments as the following:—

If a stone be dropped from the top of the mast, when a vessel is moving uniformly in any direction, and with any velocity, it will fall at the foot of the mast, just as it would if the vessel had been at rest.

If from any point a ball be let fall, and another ball be at the same instant projected forward horizontally with any velocity whatever, both balls strike the ground at the same time. Here the ball at rest and the ball in motion are acted upon by the same vertical force, namely, the force of gravity, and both are caused to pass through the same vertical space in the same time.

If a person in an open railway carriage throw a ball perpendicularly upwards, it will not fall in rear of the train, but will drop into the hands of the individual who projected it.

70. THE THIRD LAW OF MOTION.—*When pressure communicates motion to a body, the accelerating force varies as the ratio of the pressure and mass.*

Thus, if a pressure of 12 lbs. communicates motion to a mass weighing 20 lbs., and a pressure of 8 lbs. communicates motion to a mass weighing 10 lbs., the third law of motion declares that the force when 12 lbs. moves 20 lbs. : the force when 8 lbs. moves 10 lbs. :: $\frac{12}{20} : \frac{8}{10}$.

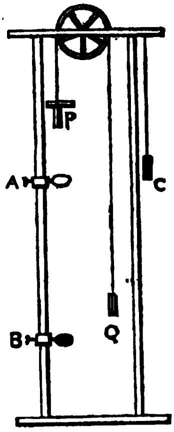
71. The force of gravity, usually represented by g, is found by experiment to be a uniform accelerating force, generating a velocity of about 32 feet per second (more accurately 32·2).

When a body falls by its own weight, the force which produces motion is the force of gravity, and hence by an easy deduction from the third law of motion, as stated above, we obtain the accelerating force when a pressure P communicates motion to a mass M; for force when P moves M : force when P moves P (that is g) :: $P \div M : P \div P$. Therefore

$$\text{force when P moves M} = g\frac{P}{M}.$$

It is under this form that the third law of motion is most conveniently verified experimentally. The mode in which this is done is explained in the following article:—

72. **Attwood's Machine.**—This consists of two pillars, one of which is graduated, supporting a pulley arranged so as to work with as little friction as possible. An open ring A and a stage B slide along the graduated pillar, and by means of screws can be fixed at any part of it. P and Q are two equal cylindrical weights connected by a cord passing over the pulley. C is a pendulum beating seconds.



P and Q, being equal, will have of themselves no tendency to motion, but if a small bar be placed upon P, P will descend with an accelerated velocity until it reach the ring A, which allowing P to pass through, but intercepting the bar, removes the cause of acceleration. P will then, in accordance with the first law of motion, move uniformly with the velocity it had acquired on reaching A; and if the stage B be so placed that P may strike it exactly one second after reaching A, the distance AB will measure the velocity generated during the interval the bar was resting on P. If then the bar be allowed to rest on P for exactly one second before reaching A, we obtain the means of determining the velocity generated in one second; that is, the measure of the accelerating force. This will always be found to be that given by the formula $f = g\frac{P}{M}$.

Thus, if the two weights are together equal to 15¾ oz. and if the bar weigh ¼ oz., then, since the pressure communicating motion is ¼ oz. and the mass moved is

16 oz. the accelerating force is affirmed by the third law, as expressed in the formula just quoted, to be $g \div 64$, or $\frac{1}{2}$; that is, it generates per second a velocity of $\frac{1}{2}$ ft. or 6 in. If then P, with the bar attached, fall for one second before reaching A (and it will be found by trial that in order to this P must move from a point just 3 in. above A), then the distance AB, over which it moves in the next second, will be found to be 6 in.

73. *To find the space described in* t *seconds when a body moves from rest under the action of a uniform accelerating force* f.

If a force be measured as explained in Art. 68 by the velocity generated in one second, it will be found by experiments with Attwood's machine, that the space described in one second is equal to one-half of the force. Thus, if the force generates a velocity of 20 ft. per second, the space described in one second will be 10 ft.; and generally, if f be a force measured by the velocity it generates per second, the space described in one second will be $\frac{1}{2}f$.

At the expiration of one second, the body will have acquired a velocity f, and, therefore, during the 2nd second the force is acting on a body in motion. By the second law of motion, each cause of motion will produce its full effect in its own direction. In the present case, both are acting in the same direction, and hence the entire space described in the 2nd second will be the sum of the separate spaces due to the velocity acquired and the action of the force. But the space due to the velocity acquired $= f$, and the space due to the action of the force $= \frac{1}{2}f$, therefore the space described in the 2nd second $= f + \frac{1}{2}f = \frac{1}{2}f \times 3$.

The velocity acquired in two seconds is $2f$. Hence, in the 3rd second we have the space due to the acquired velocity $= 2f$, and space due to the action of the force $= \frac{1}{2}f$, therefore the space described in the 3rd second $= 2f + \frac{1}{2}f = \frac{1}{2}f \times 5$.

In like manner it will be found that the space described in the 4th second $= \frac{1}{2}f \times 7$; in the 5th, $\frac{1}{2}f \times 9$; and so, generally, in the *n*th the space described $= \frac{1}{2}f \times$ the *n*th odd number.

By adding together the spaces described in the 1st and 2nd second, we obtain the space described in two seconds; adding to this the space described in the 3rd second we obtain the space described in 3 seconds, and so on. Hence,

space described in 2 seconds $= \frac{1}{2}f \times 4$,
" " 3 " $= \frac{1}{2}f \times 9$,
" " 4 " $= \frac{1}{2}f \times 16$,

and generally, if s be the space described in t seconds,

$$s = \frac{1}{2}ft^2.$$

74. *To find the relation between the space described and the velocity acquired by a body moving from rest under the action of a uniform accelerating force.*

By Art. 68, $v = ft$, and by the preceding article $s = \frac{1}{2}ft^2$. Squaring both sides of the former, and multiplying both sides of the latter by $2f$, we obtain

$$\begin{aligned} v^2 &= f^2t^2 \\ \text{and}\quad 2fs &= f^2t^2 \\ \therefore\quad v^2 &= 2fs. \end{aligned}$$

75. The three equations $v = ft$, $s = \frac{1}{2}ft^2$, and $v^2 = 2fs$, express the relation between each pair of the three quantities, viz., the space described, the velocity acquired, and the time, when a body moves from rest under the action of a uniform accelerating force. If any one of these quantities be known, either of the

other two may, by means of one or other of these equations, be found directly. When gravity is the force considered, these equations are written $v = gt$, $s = \frac{1}{2}gt^2$, and $v^2 = 2gs$.

Ex. 1. A stone falls from rest under the action of gravity, find the space described in 5 seconds.

In the formula $s = \frac{1}{2}gt^2$, make $t = 5$, then

$$s = 16{\cdot}1 \times 25 = 402{\cdot}5 \text{ feet.}$$

Ex. 2. Find the velocity which a stone will acquire by falling through 1610 feet.

In the formula $v^2 = 2gs$, make $s = 1610$, then

$$\begin{aligned} v^2 &= 64{\cdot}4 \times 1610, \\ &= 103684, \\ \therefore \quad v &= \sqrt{103684} = 322. \end{aligned}$$

Ex. 3. How long must a body fall under the action of gravity to acquire a velocity of 128·8 feet per second? In the formula $v = gt$, make $v = 128{\cdot}8$, then

$$\begin{aligned} 128{\cdot}8 &= 32{\cdot}2 \times t, \\ \therefore \quad t &= 128{\cdot}8 \div 32{\cdot}2 = 4 \text{ seconds.} \end{aligned}$$

Ex. 4. How far must a body fall under the action of gravity to acquire a velocity 96·6.

In the formula $v^2 = 2gs$, make $v = 96{\cdot}6$; then

$$\begin{aligned} (96{\cdot}6)^2 &= 64{\cdot}4 \times s \\ \therefore \quad s &= 144{\cdot}9. \end{aligned}$$

Ex. 5. A weight of 9 ounces draws a weight of 7 ounces over a fixed pulley, find the space described from rest in t seconds, neglecting the inertia of the pulley.

The pressure causing motion is $9 - 7 = 2$ ounces, and the weight moved is $9 + 7 = 16$ ounces, therefore (Art. 71),

$$\text{the accelerating force} = g \cdot \frac{2}{16} = 4 \text{ nearly.}$$

Substituting this in the general formula $s = \frac{1}{2}ft^2$, we have

$$s = 2t^2.$$

Hence, in one second the space described is 2 feet, in two seconds 8 feet, in three seconds 18 feet, and so on.

Ex. 6. A weight of 9 lbs. is drawn along a smooth horizontal table by a weight of 1 lb. hanging vertically by a string passing over a pulley at the edge of the table; find the space described from rest in 3 seconds, the velocity acquired in 4 seconds, and the space described in the 5th second.

The pressure causing motion is 1 lb., the weight moved is $9 + 1 = 10$ lbs. Therefore

$$\text{accelerating force}, = g.\frac{1}{10} = 3{\cdot}2,$$

space described in 3 seconds $= 1{\cdot}6 \times 9 = 14{\cdot}4$ ft.
velocity acquired in 4 seconds $= 3{\cdot}2 \times 4 = 12{\cdot}8$.

The space described in the 5th second is $\frac{1}{2}f$ multiplied by the 5th odd number, or 9; $\therefore$ space described in 5th second $= 1{\cdot}6 \times 9 = 14{\cdot}4$ feet.

EXAMPLES.

(In the following examples take $g = 32$.)

If a body fall from rest under the action of gravity,

Find 1. The space described in the 21st second.
Ans. 656 ft.

„ 2. The velocity acquired in 8 seconds.
Ans. 256.

„ 3. The space described in 6 seconds.
Ans. 576 ft.

„ 4. The time of acquiring a velocity 160.
Ans. 5 sec.

„ 5. The space described in acquiring a velocity 224.
Ans. 784 ft.

Find 6. The time of describing 400 ft.
Ans. 5 sec.

,, 7. The velocity acquired in describing 576 ft.
Ans. 192.

8. A weight of $8\frac{1}{2}$ lbs. descends drawing up another of $7\frac{1}{2}$ lbs. over a fixed pulley, find the space described in 5 seconds. Ans. 25 ft.

9. A weight of 13 lbs. is drawn along a smooth horizontal table by a weight of 3 lbs. find the time of describing 147 ft. Ans. 7 sec.

76. *If a body be projected with a given velocity* V, *and be acted on in the same direction by a uniform accelerating force* f, *to find the velocity acquired and the space described in a given time* t.

From the second law of motion, it follows that the velocity will equal the velocity of projection + the velocity generated by the force; and, therefore, if v denote the velocity required,

$$v = V + ft.$$

And similarly, the space described = the space due to the velocity of projection + the space due to the action of the force. The space described by a uniform velocity V in t seconds $= Vt$, and the space due to the action of a uniform accelerating force in t seconds $= \frac{1}{2}ft^2$. Therefore

$$\therefore \quad s = Vt + \tfrac{1}{2}ft^2.$$

77. If a body be projected with a given velocity V in a direction opposite to that in which the accelerating force acts, the expressions deduced as in the preceding article will be

$$v = V - ft$$
$$s = Vt - \tfrac{1}{2}ft^2.$$

Squaring both sides of the first equation, we have

$$\begin{aligned} v^2 &= V^2 - 2Vft + f^2t^2 \\ &= V^2 - 2f(Vt - \tfrac{1}{2}ft^2) \\ &= V^2 - 2fs. \end{aligned}$$

78. *To find the height to which a body will rise when projected vertically with a given velocity V.*

Here the direction of V is opposite to that of the force of gravity or g, and hence if v denote the velocity acquired when the body has described the space s, by the preceding article,

$$v^2 = V^2 - 2gs.$$

But when the body has attained the highest point, its velocity at that instant will $= 0$, and therefore if $s =$ the height of this point,

$$0 = V^2 - 2gs.$$

$$\therefore \quad 2gs = V^2.$$

By comparing this result with the expression $v^2 = 2gs$ in Art. 74, we see that the height to which the body will rise is the same as the distance through which it must fall to acquire the velocity of projection.

79. *To find the time during which a body will rise when projected vertically with a given velocity* V.

Since at the instant of attaining the greatest height $v = 0$, then (Art. 77),

$$\begin{aligned} V - gt &= 0 \\ \therefore \quad gt &= V \\ \therefore \quad t &= \frac{V}{g}. \end{aligned}$$

From the general formula $v = gt$, it follows that $\frac{V}{g}$ is also the time during which a body must fall to acquire the velocity V, and hence from this and the preceding section we see that on its return to the point whence it was projected, the body will have a velocity equal to the velocity of projection, and that the times of descent and ascent will be equal.

Ex. 1. A body is projected vertically upwards with a velocity of 100 feet per second, to find how high it will rise.

By Art. 78, $2gs = V^2$; therefore, in this case,

$$s = \frac{10000}{64 \cdot 4} = 155 \cdot 28.$$

Ex. 2. A body is projected vertically upwards with a velocity of 161 feet per second, to find how long it will continue to ascend.

By Art. 79, $t = \frac{V}{g}$, therefore in this case,

$$t = \frac{161}{32 \cdot 2} = 5 \text{ seconds.}$$

Ex. 3. A stone is projected vertically upwards, and returns to the same spot after an interval of 12 seconds, find the velocity of projection, and the height to which the body has risen.

The velocity of projection is equal to the velocity acquired by the body during the time of descent, or 6 seconds; and therefore, if V be the velocity of projection, since $v = gt$,

$$V = 32 \cdot 2 \times 6 = 193 \cdot 2 \text{ feet per second.}$$

The height to which the body has risen is equal to the distance through which it falls from rest during 6 seconds, that is,

$$\begin{aligned} \text{the height required} &= \tfrac{1}{2}g \times 6^2 \\ &= 16 \cdot 1 \times 36 = 579 \cdot 6 \text{ feet.} \end{aligned}$$

Ex. 4. A ball is projected vertically with a velocity of 160 feet per second, and two seconds afterwards another ball is projected in the same direction with a velocity 224, when, and at what height, will the balls meet?

Let the balls meet x seconds after the projection of the second ball, then $x + 2$ is the time the first ball has been moving.

The space described by the first ball in $x + 2$ seconds is (Art. 77),

$$160\,(x + 2) - 16\,(x + 2)^2.$$

Similarly, the space described by the second ball in x seconds is

$$224x - 16x^2.$$

But since the balls meet, these two quantities must be equal; ∴

$$\begin{aligned} 224x - 16x^2 &= 160\,(x + 2) - 16\,(x + 2)^2 \\ \therefore\quad 128x &= 256 \\ \therefore\quad x &= 2, \end{aligned}$$

or the balls meet 2 sec. after the projection of the second ball. To find the height, substitute 2 for x in either of the expressions for the space described by the balls; *e.g.*, $224x - 16x^2$,

$$\text{height} = 224 \times 2 - 16 \times 4 = 384.$$

EXAMPLES.

1. A ball is projected vertically upwards with a velocity of 80 feet per second; how high will it rise in 2 seconds? Ans. 96 feet.

2. A ball is projected vertically upwards with a velocity of 192 feet per second; after what interval will it return to the point of projection? Ans. 12 sec.

3. A ball is projected vertically upwards with a velocity of 80 feet per second; what is its greatest elevation? Ans. 100 feet.

4. A ball is projected vertically downwards with a velocity of 70 feet per second; what is the space described in 6 seconds? Ans. 996.

5. A ball is projected horizontally with a velocity of 60 feet per second; what is the distance of the ball from the point of projection at the end of 5 seconds? Ans. 500 feet.

6. A ball is projected vertically upwards with a velocity of 1000 feet per second, and 3 seconds afterwards another ball is projected in the same direction with a velocity 1856; at what time after the projection of the second ball will the balls meet?

Ans. 3 sec.

7. If in the preceding question the balls had been projected downwards, when would they have met?

Ans. $4\frac{13}{95}$ sec. after the projection of the second ball.

8. A ball is projected horizontally with a velocity of 40 feet per second; from the same point and at the same instant a second ball is let fall; what distance will the balls be apart in 8 seconds? Ans. 320 feet.

9. Two balls are projected at the same time from the same point, one horizontally with a velocity 60, and the other vertically downwards with a velocity 20; what will be the distance of the balls from each other in 5 seconds? Ans. 316·23 feet nearly.

CHAPTER V.

ON THE FUNDAMENTAL PROPERTIES OF FLUIDS.

80. A fluid is a body, all of whose parts can move freely amongst themselves. Motion consequently can be caused amongst the particles of a fluid body, by the application of the slightest conceivable force.

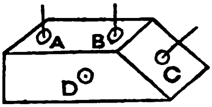
81. The science which treats of the equilibrium of forces acting upon fluid bodies is termed Hydrostatics.

82. Fluids are divided into liquids and aeriform fluids. An aeriform fluid is distinguished from a liquid by the existence of an expansive or repulsive force amongst its particles, in consequence of which they tend in a greater or less degree to move off from one another.

83. When a fluid is at rest, any portion of it may be supposed to become solid, without disturbing the equilibrium.

Since all the particles of the fluid are at rest, it is clear that they will not less be so, if any number of them be rigidly connected with each other, and so deprived of the power of relative motion.

84. *When pressure is communicated to a fluid mass in equilibrium, it is transmitted equally and in all directions.*



In the sides of a closed vessel of any shape, let a number of apertures of equal area be made, and let these apertures be supplied with pistons exactly fitting them. Let the vessel be filled with water, and let the pistons be maintained by some mechanical contrivance in their respective positions.* If any one of the pistons be pressed in with a force P, it is found that all the other pistons experience a similar pressure, and that a corresponding force P must be applied to each of them, in order to retain them in their places. Thus the pressure communicated to the fluid has been transmitted in all directions, for all the pistons experience it, whatever their position and whatever the shape of the vessel; and the transmitted pressure is equal to that communicated.

These results are the same, whatever the number of the pistons, and wherever placed. It follows, therefore, that every portion of the surface of the containing vessel, equal in area to that of the piston, experiences a pressure P. Thus, if the area of a piston is one square inch, and a pressure of 1 lb. is exerted upon it, a corresponding pressure of 1 lb. is experienced by every square inch of the surface of the containing vessel.

85. If A and B, the areas of whose lower surfaces are a and b respectively, be two pistons fitted into the upper surface of a vessel filled with fluid, and if Q be the pressure experienced by B when a pressure P is exerted upon A, then

$$Q : P :: b : a.$$

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* For the reason of this provision, see Art. 91.

For in consequence of the pressure upon the piston A, a pressure P is transmitted to every portion of the surface of the containing vessel, whose area equals a, and therefore to every unit of area a pressure is transmitted equal to $P \div a$.

Since the lower surface of B contains b units of area, the whole pressure experienced by B is $Pb \div a$. But by the hypothesis, Q denotes this pressure, therefore $Q = Pb \div a$, that is,

$$Q : P :: b : a,$$

$$\text{or} \quad \frac{\text{pressure on B}}{\text{pressure on A}} = \frac{\text{area of B}}{\text{area of A}}.$$

As an illustration, let the lower surface of the piston A be a circle whose diameter is 1 inch, and that of B a circle whose diameter is 1 foot, then

$$\frac{\text{area of B}}{\text{area of A}} = \frac{12^2}{1^2} = 144,$$

and therefore

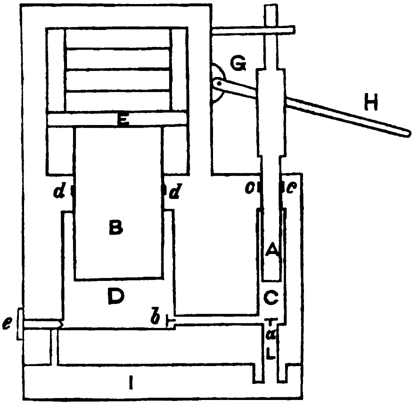
$$\frac{\text{pressure on B}}{\text{pressure on A}} = 144,$$

$$\text{or} \qquad \text{pressure on B} = 144 \times \text{pressure on A};$$

so that if a weight of 1 lb. rest upon the piston A, a weight of 144 lbs. must be placed upon the piston B in order to maintain equilibrium; and conversely, if 144 lbs. press on the piston B, the equilibrium may be maintained by a pressure of 1 lb. on A.

86. **Bramah Press.** This powerful machine, invented by Mr. Bramah, is an interesting application to practical purposes of the characteristic property of fluids.

C and D are two cylindrical vessels connected with each other by means of the pipe *ab*. A and B are two solid pistons working in water tight collars *cc*, *dd*. The piston B, the diameter of which is much greater than that of A, supports a plate E, upon which the substance to be pressed is placed. A is capable of being worked up and down by a lever G H, having its fulcrum at G. L is a pipe leading into a reservoir I, *a* is a valve opening upwards, and *b* a valve opening into the vessel D.



Let the vessels C D be supposed to be filled up with water, when the pistons are in the position represented in the figure. Let A be forced down with a pressure P, then by the preceding article B is forced up with a pressure $= P \times \frac{\text{area of B}}{\text{area of A}}$.

If B yield to this pressure, the piston A will descend, and a portion of the water in C will be forced into the vessel D. The return of the fluid will be prevented by the valve *b*. If the piston A be now raised, fresh water will be pumped up into the vessel C from the reservoir I, and if A be again forced down with a pressure P, B will be again forced up with the same pressure as before. This process may be continued as long as the substance yields to the pressure exerted upon it.

The pressure upon B may at any time be removed by unscrewing the plug *e*, by which the water is allowed to flow back into the reservoir.

87. The safety valve is another important application of the same principle. A portion of the boiler of a steam-engine, whose area contains *a* square inches, is furnished with a valve opening outwards. The valve is so contrived as to admit of being loaded with certain weights. If a weight of *a*P lbs. be placed on the valve, it is pressed down with a pressure of P lbs. to the square inch. If then at any time the pressure of the steam be greater than P lbs. to the square inch, the valve will be forced open, and the steam will escape; and since the pressure of the steam is the same on every inch of the surface of the containing vessel, no portion of the boiler will experience a pressure greater than P lbs. to the square inch. So that if P be less than the pressure per square inch which the weakest portion of the boiler can bear, the boiler can never burst.

88. All the particles of a fluid, equally with those of any solid body, are subject to the action of gravity. Any portion, however small, of a fluid is drawn towards the earth with a certain degree of force, that is, is possessed of weight (Art. 29). The total pressure, therefore, of a fluid at rest upon any surface in contact with it may be, and most commonly is, the sum of two dif-

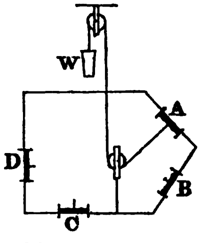
ferent pressures, the one a pressure transmitted simply by the fluid, and the other a pressure arising from the weight of the fluid itself. The former, as we have seen, is the same upon every unit of a surface in contact with the fluid. The latter, it will be seen hereafter, varies with the position of the surface. When the latter is known in any case, it is only necessary to add to it the amount of transmitted pressure corresponding to the area of the surface, and the total pressure is found. If then we can determine the pressure of a fluid upon any surface arising from the action of gravity upon its particles, upon the supposition that no external pressure is communicated to the fluid, the problem is completely solved.

89. The pressure of a fluid at rest upon any plane, must be in a direction perpendicular to the plane.

For the only force opposed to the pressure of the fluid on the plane, is the resistance of the plane itself, and since there is equilibrium, these two forces must be equal and opposite. The resistance of the plane is perpendicular to the plane, this consequently must be the direction of the pressure of the fluid.

90. *The pressure of a fluid upon any plane in contact with it is equal to the weight of a column of the fluid whose base is the area of the given plane, and whose height is the depth of the centre of gravity of the plane below the surface of the fluid.*

Let A, B, C, D be moveable pistons, with plane surfaces, fitted into the sides of any vessel. If the vessel be filled with water, or any other fluid, the pressure of the fluid will force these pistons outwards. The amount of this pressure may be learnt by finding the force necessary to keep the pistons in their places. As the pressure acts at right angles to the surface of the piston, this force must also act at right angles to the piston. Let a cord be attached to any one of these pistons, as at A, and pass over pulleys so placed that the cord forms a right angle with the surface of the piston. Let W be the weight which must be attached to the other extremity of the cord in order to keep the piston in its place. It will be found that W is equal to the weight of a column of the fluid whose base is the area of the piston A, and whose height is the depth of the centre of gravity of the surface of the piston below the surface of the fluid. Thus, if the area of the piston be 6 square inches, and the depth of its centre of gravity be 8 inches, W will be equal to the weight of 6 × 8, or 48 cubic inches of the fluid. By similar experiments with other pistons, such as B, C and D, whose surfaces are differently inclined, it will be found that the result is independent of the inclination of the surface; so that if the surface be the same, and the depth of its centre of gravity be the same, the pressure of the fluid is the same, whatever the inclination of the given surface may be.



Ex. 1. To find the pressure on a rectangular plane 10 inches by 4, when immersed in water so that its centre of gravity is at a depth of 20 inches.

The area of the plane is 10 × 4, or 40 square inches. Hence the pressure on the plane is the weight of 40 × 20, or 800 cubic inches of water.

The weight of a cubic foot of pure water is 1000 oz.

avoirdupois, and therefore the weight of a cubic inch is $1000 \div 1728$ oz.

$$\therefore \text{ Pressure required} = \frac{800 \times 1000}{1728}$$
$$= 462{\cdot}96 \text{ oz.}$$

Ex. 2. To compare the pressures on the bottom and side of a cubical vessel filled with any fluid.

Let a be the length of the edge of the cube, and w the weight of a cubic unit of the fluid. The area of the bottom of the vessel is a^2, and the depth of its centre of gravity is a, $\therefore$ the pressure on the bottom $= wa^3$.

The area of the side of the vessel is a^2, and the depth of its centre of gravity is $\frac{1}{2}a$, $\therefore$ the pressure on the side is $\frac{1}{2}wa^3$. Hence the pressure on the side of the vessel is one-half of that on the bottom.

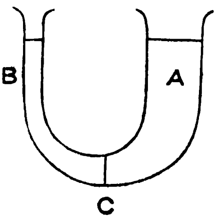
91. If the student will now revert to Art. 84, he will see why it was necessary to insert the provision that the pistons be maintained in their position by some mechanical contrivance. For if the pistons (as is the case with the pistons C, D) be situated any where except upon the uppermost surface of the vessel, they will experience a pressure greater or less according to their distance below the surface of the fluid, and will, in consequence of this pressure, be forced out if not prevented.

The pistons A and B, which press upon the uppermost surface of the fluid, experience no pressure from the weight of the fluid itself; whatever force, therefore, is impressed upon them is transmitted undiminished to every part of the fluid. But if a pressure be applied elsewhere, at C for instance, and no other force act upon the piston, then a part of this pressure will be employed in counterbalancing the pressure of the fluid upon C, and the remainder only will be the pressure transmitted by the fluid. Hence, if a be the area of any piston, h the distance of its centre of gravity below the surface of the fluid, and w the weight of a cubic unit of the fluid, and if aP be the pressure exerted upon the piston, the pressure transmitted to every corresponding area is $a\mathrm{P} - wah$, or the pressure transmitted to every unit of area is $\mathrm{P} - wh$.

92. Since the pressure of a given fluid upon a given horizontal plane, depends only upon the depth of the plane beneath the surface of the fluid, it follows that, if the bottoms of any number of vessels of various shapes be of equal area, and the same fluid be poured to the same height in all, the pressure upon the bottom of each vessel will be the same, however different the quantity of fluid in each may be. If the bottoms of the vessels be moveable, it will be found by experiment that the same force is necessary in each case to retain them in their respective positions.

93. *If a fluid be poured into any one of a number of open vessels having a free communication with each other, the fluid will rise to the same height in all.*

Let A and B be any two such vessels, having the lowest point at C.



Let a thin vertical plate of the fluid passing through C become rigid. Let a be the area of this plate, h the distance of its centre of gravity below the surface of the fluid in A, and h' its distance below the surface in B.

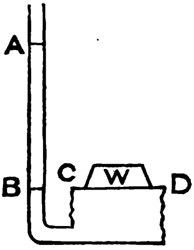
Let w be the weight of a cubic unit of the fluid. Then the pressure exerted on the one side of the plate by the fluid in A is wah, and that exerted on the other side by the fluid in B is wah'. But since there is equilibrium, these pressures must be equal; therefore

$$wah = wah'$$

or
$$h = h'.$$

94. Hydrostatic Bellows. AB is a narrow tube communicating with a vessel formed by uniting together two pieces of wood by some flexible and waterproof substance. If water be poured down the tube it will enter the vessel, and raise a large weight W placed upon CD.

When equilibrium exists, let A be the surface of the water in the tube, and let B lie in the same horizontal plane with CD. Let w be the weight of a cubic unit of water, and let h be the depth of B or CD below A.



The pressure upon each unit of area in CD is wh. Therefore, if b = number of units in the area of CD, the total pressure on CD = wbh, and since there is equilibrium, this must equal the weight supported; therefore

$$W = wbh.$$

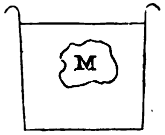
If a = area of the horizontal section of the pipe at B,

$$\text{weight of fluid in AB} = wah,$$

$$\therefore \qquad W = \frac{b}{a} \times \text{weight of fluid in AB}.$$

95. *To find the resultant of the pressure of a fluid on the surface of a solid, wholly or partially immersed in it.*

Let any portion M of any fluid at rest become solid, and since the equilibrium is not thereby disturbed, the forces acting upon M must be in equilibrium. The only forces acting upon M are the weight of M acting vertically downwards at the centre of gravity of M, and the pressures of the fluid upon the surface of M. Hence the resultant of these pressures must be equal and opposite to the weight of M acting at its centre of gravity. But the fluid will exert the same pressure upon the surface of any other body of the same form as M occupying its place. Hence the resultant of the pressure of a fluid on the surface of a solid immersed in it, is equal to the weight of the fluid displaced, and acts vertically upwards through the centre of gravity of the fluid displaced.



96. If any body hang freely by a cord, we have seen (Art. 32) that the centre of gravity lies in the vertical line drawn through the point of suspension, that is, in the line of the cord; and the tension in the cord equals the weight of the body.

If such a body be wholly immersed in a fluid, it has been shewn in the preceding article, that it will be pressed upwards by a force equal to the weight of the fluid displaced, which, in this case, is the weight of a portion of the fluid equal to the bulk of the body; and this pressure is directly opposite to the weight of the body; therefore the tension in the cord will be the weight of the body diminished by the weight of an equal bulk of the fluid.

97. *To determine the conditions of equilibrium of a floating body.*

If a body floating in a fluid be at rest, the forces acting upon it are the weight of the body, which acts at its centre of gravity, and the resultant pressure of the fluid which acts at the centre of gravity of the fluid displaced.

The necessary conditions of equilibrium are, that these forces be *equal* and *opposite*. The two conditions, therefore, are,

First, That the weight of the body be equal to that of the fluid displaced; and,

Secondly, That the centres of gravity of the body and the fluid displaced be in the same vertical line.

As a body cannot displace a quantity of fluid greater than its own bulk, if the weight of the body be greater than that of an equal bulk of the fluid, the body cannot float in that fluid. If the weight of the body be exactly equal to that of an equal bulk of the fluid, the body will have in no position any tendency either to rise or to sink, provided only it be entirely covered by the fluid.

EXAMPLES.

1. Two circular pistons, whose diameters are 2 and 10 inches respectively, are fitted into the upper surface of a vessel filled with fluid, if a pressure of 10 lbs. be applied to the smaller piston, what is the pressure on the larger piston?* Ans. 250 lbs.

2. The diameter of the large piston in a Bramah press is 20 inches, of the small $\frac{1}{4}$ inch, the lever is 2 feet long, and is attached to the small piston rod at a point 2 inches from the fulcrum, what is the mechanical advantage of the press? Ans. 76800.

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* The area of a circle is nearly 22-7ths of the *square* of its radius. The content of a globe is 4-3rds of 22-7ths of the *cube* of the radius.

3. If the side of a vessel be a rectangle, shew that the pressure upon it, when the vessel is filled with any fluid, is four times as much as when the vessel is only half filled.

4. Find the pressure upon a rectangular plane 12 in. by 5, when immersed in water, so that its centre of gravity is at a depth of 24 inches. Ans. $833\frac{1}{3}$ oz.

5. The diameter of the pipe of a hydrostatic bellows is 2 inches, and that of the plate is 12 inches, a weight of 250 lbs. is placed upon the plate, what will be the height of the water in the pipe?

Ans. $61\frac{1}{11}$ in. nearly.

6. In the preceding, if the height of the water in the pipe be 28 in. what weight will be supported on the plate? Ans. 114·6 lbs. nearly.

7. A cubic foot of fir weighs 40 lbs., to what depth will it be immersed when floating in water?

Ans. 7·68 in.

8. If a cubic foot of oak, when floating in water, be immersed to the depth of 9 inches, what is its weight?

Ans. 46 lbs. 14 oz.

9. A globe 6 inches in radius, when floating in water is half immersed, what is its weight? Ans. $261\frac{18}{21}$ oz.

CHAPTER VI.

ON SPECIFIC GRAVITY.

98. The specific gravity of any substance is the ratio of the weights of equal bulks of that substance, and of a certain standard substance.

The standard for solids and liquids is distilled water at a temperature of 60°.

The standard for gases is pure atmospheric air at a temperature of 60°.

Hence, if the specific gravity of any solid or liquid be 2, the weight of a cubic inch of the substance is twice the weight of a cubic inch of water. Or, generally, if s be the specific gravity of any substance, and w the weight of a cubic unit of the standard, the weight of a corresponding unit of the substance is sw.

The absolute weight of any substance may thus be easily found from a table of specific gravities.

For example, to find the weight of a cubical block of marble, whose side is 10 feet; the specific gravity of the marble being 2·7.

A cubic foot of pure water, at a temperature of 60°, weighs 1000 ounces avoirdupois.

$$\therefore \text{ weight of a cubic foot of marble} = 2{\cdot}7 \times 1000.$$
$$= 2700 \text{ oz.}$$
$$= 168{\cdot}75 \text{ lbs.}$$

But the block contains 10^3, or 1000 cubic ft. of marble. Therefore,

$$\text{weight of the block} = 168{\cdot}75 \times 1000,$$
$$= 168750 \text{ lbs.}$$
$$= 75{\cdot}334 \text{ tons.}$$

99. The Hydrostatic Balance is, in its simplest form, a common pair of scales with a fine thread attached to the under surface of one of the scale pans. By this arrangement, any solid substance can be weighed, both in the ordinary way, and also by means of the thread when immersed in any fluid.

100. *To find the specific gravity of a body heavier than water.*

Let W be the weight of the body in air, and w its weight in water. It has been seen (Art. 96,) that w differs from W by the weight of a quantity of water equal to the bulk of the body immersed. Hence,

wt. of water equal in bulk to the given body $= W - w$.

$$\therefore \text{ sp. gr. of the given body} = \frac{W}{W - w}.$$

101. *To find the specific gravity of a body lighter than water.*

Let A be the given body, and B any other body, such that, when attached to A, both will sink in pure distilled water.

Let W be the weight of A + B in air, and w their weight in water. Let W' be the weight of B in air, and w' its weight in water. Then:

$$\left.\begin{array}{c}\text{weight of water equal}\\ \text{in bulk to}\end{array}\right\}A + B = W - w$$

$$\text{,, } \quad \text{,, } \qquad B = W' - w'$$

$$\therefore \quad \text{,, } \quad \text{,, } \qquad A = W - w - (W' - w')$$

$$\text{But} \quad \text{weight of A in air} = W - W'$$

$$\therefore \qquad \text{sp. gr. of A} = \frac{W - W'}{W - w - (W' - w')}$$

102. *Giving the weights and specific gravities of the components, required the specific gravity of the compound.*

Let the compound be formed by the mixture of a body A, whose weight is W and sp. gr. s, with a body B whose weight is W' and sp. gr. s'. Then, by the definition,

$$s = W : \text{weight of water equal in bulk to A,}$$

$$\text{and } s' = W' : \text{weight of water equal in bulk to B.}$$

$$\therefore \qquad \text{weight of water equal in bulk to A} = \frac{W}{s}$$

$$\text{and} \qquad \text{weight of water equal in bulk to B} = \frac{W'}{s'}$$

$$\therefore \qquad \left.\begin{array}{r}\text{weight of water equal in}\\ \text{bulk to the compound}\end{array}\right\} = \frac{W}{s} + \frac{W'}{s'}$$

$$\text{But} \quad \text{weight of compound in air} = W + W'$$

$$\therefore \qquad \text{sp. gr. of compound} = \frac{W + W'}{\dfrac{W}{s} + \dfrac{W'}{s'}}$$

It is assumed here that the volume of the compound is the sum of the volumes of the component bodies. This, however, is not always the case, as very frequently the volume of the compound is affected by the chemical combination of the components.

103. If the bulks of the components are known and not their weights, the specific gravity of the compound may be found as follows. Let V, V′ be the number of cubic units in A and B respectively. Let w be the weight of a unit of water. Then

$$\begin{aligned} \text{weight of } A &= Vsw \\ \text{,,}\quad B &= V's'w, \\ \text{,,}\ A + B &= Vsw + V's'w. \end{aligned}$$

The weight of water equal in bulk to A + B, that is, containing V + V′ cubic units, is $(V + V')w$.

$$\therefore \quad \text{sp. gr. of compound} = \frac{Vsw + V's'w}{(V + V')w} = \frac{Vs + V's'}{V + V'}$$

104. *To determine the specific gravity of a fluid by means of the specific gravity bottle.*

The specific gravity bottle is simply a small flask fitted with a ground stopper.

Let w be the weight of the flask, x its weight when filled with the given fluid, and y its weight when filled with distilled water. Then $x - w =$ the weight of the given fluid contained in the flask, and $y - w =$ the weight of a similar volume of distilled water.

$$\therefore \quad \text{sp. gr. of the fluid} = \frac{x - w}{y - w}.$$

Specific gravity bottles are very frequently made so as to contain exactly 1000 grains of pure distilled water. Then, if x and w are known in grains,

$$\text{sp. gr. of the fluid} = \frac{x - w}{1000}.$$

105. *To determine the specific gravity of a fluid by weighing a solid body in it.*

Let w be the weight of the solid body, x its weight when suspended in the given fluid, and y its weight when suspended in distilled water. Then (Art 96,) $w - x$ is the weight of a quantity of the given fluid equal in bulk to the solid body, and $w - y$ is the weight of a similar volume of distilled water.

$$\therefore \quad \text{sp. gr. of the given fluid} = \frac{w-x}{w-y}.$$

Or, if s be the specific gravity of the solid, then since (Art. 100) $s = \dfrac{w}{w-y}$.

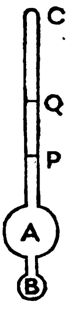
$$\text{sp. gr. of the given fluid} = s.\frac{w-x}{w}.$$

106. *To determine the specific gravity of a fluid by means of the common hydrometer.*

The common hydrometer consists of a small hollow sphere A, to which there is attached, on one side, a slender graduated stem, and on the other a smaller sphere B, made of such a weight as to allow the whole instrument to float, with the sphere A entirely immersed.

When placed in distilled water, let the instrument sink as far as P, and when placed in the fluid, whose specific gravity is sought, let it sink to Q.

Let W be the weight of the hydrometer, and V its volume. Let a be the area of the section of the stem. Let CP $= x$, and CQ $= y$.



By Art. 97, when a body floats, the weight of the body is equal to that of the fluid displaced.

The quantity of water displaced is equal to the volume of the hydrometer, minus the part CP, that is, is equal to V $-$ ax. Therefore, W = the weight of a quantity of water, whose volume is $V - ax$.

$$\therefore \quad \frac{W}{V - ax} = \text{weight of a cubic unit of water.}$$

Also: W = the weight of a quantity of the given fluid whose volume is $V - ay$.

$$\therefore \quad \frac{W}{V - ay} = \text{weight of a cubic unit of the given fluid.}$$

$$\text{And} \therefore \text{ sp. gr. of the given fluid} = \frac{W}{V - ay} : \frac{W}{V - ax}$$

$$= \frac{V - ax}{V - ay}.$$

TABLE OF SPECIFIC GRAVITIES.

Platinum (forged)..	21·4	Plate Glass	2·9
Gold	19·3	Marble	2·7
Mercury (congealed)	15·6	Green Glass	2·6
——— (fluent)....	13.6	Flint (black)	2·6
Lead	11·3	Portland Stone	2·5
Silver	10·5	Brick	2·0
Copper	8·9	Ivory	1·8
Steel	7·8	Lignum Vitæ	1·3
Iron (forged)	7·7	Ebony	1·2
Tin	7·3	Mahogany ·64 to....	1·1
Iron cast at Carron..	7·2	Beech	.85
Zinc compressed....	7·2	Oak (English)	·75
—— in its common state	6·9	Scotch Fir	.69
Potassium	·97	Spruce Fir from Canada	·52
Sodium	·86	Cork	·24

EXAMPLES.

1. What is the weight of a block of gold 8 in. long, 6 in. wide, and 4 in. deep?

Ans. 134 lbs. $0\frac{4}{9}$ oz. avoirdupois.

2. What is the weight of a cast iron globe whose radius is 12 in.? Ans. 16 cwt. 93 lbs. 11$\frac{3}{7}$ oz.

3. A body weighs 340 grains in air, and 102 grains in water, what is its specific gravity? Ans. 1·429.

4. A lump of iron weighing 2 lbs. 4 oz in air, and 1 lb. 15 oz. in water, is attached to a piece of cork, the two together weigh 2 lbs. 10 oz. in air and 12 oz. in water, what is the sp. gr. of the cork? Ans. ·24

5. If 12 lbs. 1 oz. of gold be mixed with 11 lbs. 2 oz. of copper, what is the sp. gr. of the compound? Ans. 12·37.

6. If 20 cubic inches of tin be mixed with 30 inches of copper, what is the sp. gr. of the compound? Ans. 8·26.

7. A cube, whose edge measures 12 inches, sinks in water to a depth of 8 inches, but sinks in a given fluid to the depth of 10 inches, what is the sp. gr. of the fluid? Ans. ·8.

8. A body, whose weight is 300 grains, sinks to the same depth in water and spirits of wine, when loaded 80 and 17 grains respectively, what is the sp. gr. of the spirits of wine? Ans. ·834.

9. A cylindrical piece of cork, whose length is 20 inches, floats upright in water, to what depth will it be immersed? Ans. 4·8 in.

10. To what depth will the same piece of cork be immersed in a fluid whose sp. gr. is ·72? Ans. 6$\frac{2}{3}$ in.

CHAPTER VII.

ON ATMOSPHERIC PRESSURE.

107. The atmosphere surrounding the earth, being a fluid acted on by gravity, presses upon all bodies immersed in it, in accordance with the general laws of fluid pressure.

The pressure of the atmosphere upon any body will consequently vary with the depth of that body below its highest surface. As, however, the dimensions of any ordinary body upon the surface of the earth are inconsiderable, when compared with the height of the atmosphere, this pressure may, without any sensible error, be regarded as the same at all points in any such body.

It is found by experiment that the pressure which the atmosphere exerts upon a body, on or near the surface of the eath, is the same as that exerted by a column of water 32 feet in height, or by a column of mercury 30 inches in height, that is, it presses with a weight of about 15 lbs. upon every square inch of surface. That we are not, in ordinary circumstances, cognizant of this pressure arises from the characteristic property of fluids, in accordance with which they press in all directions upon bodies immersed in them; so that, if the upper surface of a body, subject to the pressure of the atmosphere, experience a downward pressure of 15 lbs. to every square inch, the under surface experiences a corresponding upward pressure. If, however, we destroy this equilibrium, by removing or diminishing the pressure of the atmosphere upon one side of any body, we are immediately made sensible of the pressure which is exerted upon the opposite side. For instance, if a plate of glass be placed upon the top of a cylindrical receiver connected with an air pump, and the air be exhausted from the receiver, the plate of glass will be pressed down with a considerable force, and if not strong enough will be broken in pieces.

108. *When the atmosphere presses upon any point of a fluid mass, to determine the amount of transmitted pressure.*

If the atmosphere presses upon the highest surface of the fluid mass, then it is evident that the transmitted pressure is equal to the pressure of the atmosphere.

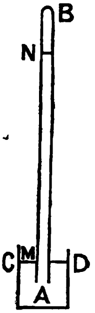
If the atmosphere presses at a point in the fluid mass below the highest surface, then a portion of the atmospheric pressure will be employed in counterbalancing the outward pressure of the fluid itself, and the remaining portion only of the atmospheric pressure will be transmitted by the fluid. Let h be the distance below the highest surface of the fluid mass of the point at which the atmosphere presses upon it, and let w be the weight of a cubic unit of the fluid, then as seen in Chapter V. the fluid itself will exert at this point an outward pressure on every square unit $= wh$. Then, if $Æ$ denote the pressure exerted by the atmosphere upon a square unit, the transmitted pressure will be

$$Æ - wh$$

for every unit of surface in the fluid mass.

If the point at which the atmosphere presses upon the fluid mass be so taken, that *wh* is greater than Æ, then it is plain that equilibrium cannot subsist, but motion will ensue amongst the particles of the fluid, until the distance becomes such that $wh = Æ$.

109. The Barometer. This instrument is constructed in the following manner:—A glass tube closed at one end is filled with mercury, and a finger being placed over the open end, is inverted in an open vessel of mercury.



Let AB be such a tube, let CD be the surface of the mercury in the vessel, and let MB be more than 30 inches. Then, since CD is more than 30 inches below B, the highest surface of the fluid mass, the pressure of the fluid upon any square unit in the surface CD will be greater than the atmospheric pressure. The mercury will consequently fall in the tube. Let N be the point at which it rests, then if Æ be the atmospheric pressure, and *w* the weight of a cubic unit of mercury,

$$Æ = w.MN.$$

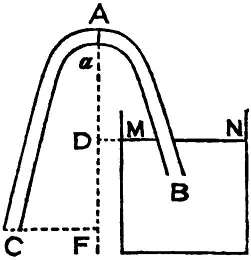
The height of mercury in the tube will accordingly vary as the atmospheric pressure varies; will rise as the atmospheric pressure increases, and fall as it decreases.

A graduated scale is attached to the tube AB, by means of which the changes in the height of the mercury in the tube can be observed and measured.

110. The Siphon. The siphon is a bent tube, open at both ends, and having one leg shorter than the other. If such a tube be filled with any fluid, and placed with

the extremity of its shorter leg in a vessel containing the same fluid, the fluid will flow out through the longer leg.

Let ABC be the siphon filled with fluid, and having B the extremity of its shorter leg AB, placed below MN, the surface of the fluid in the vessel. Let A be the highest point of the siphon, and through A draw the vertical line AF. Let MN produced meet this vertical line in D, and let CF be the horizontal line drawn through C.



Let A*a* be a thin vertical film of the fluid passing through A. Since each side of this film is similarly situated with respect to A, the highest point of the fluid mass, the pressure arising from the weight of the fluid will be the same on each side. These pressures, therefore, are in equilibrium, and do not cause the motion of the fluid.

But if Æ be the atmospheric pressure upon a square unit, and *w* be the weight of a cubic unit of the fluid, then, since the atmosphere presses upon the surface MN at a distance AD below the highest point of the fluid mass, there is transmitted up the leg BA, by Art. 108, a pressure upon every square unit, equal to Æ — *w*.AD.

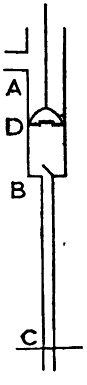
Similarly, since the atmosphere presses upon the surface of the fluid mass at C, there is transmitted up CA a pressure upon every square unit equal to Æ — *w*.AF.

Hence, the one side of the film A*a* will experience a pressure upon every square unit equal to Æ — *w*.AD, and the other side a pressure equal to Æ — *w*.AF.

But since AD is less than AF, Æ — w.AD is greater than Æ — w.AF; the film of fluid will therefore be forced down the leg AC. The same will happen to every film that may in succession occupy the position Aa, the fluid consequently will flow out at C, so long as there remains in the vessel any fluid above B.

It has been assumed in the preceding, that the height AD is such, that w.AD is less than Æ. If AD be so great that w.AD is greater than Æ, then, as seen in the preceding articles, the fluid will sink in the leg AB to some level below the bend, and the siphon consequently will cease to act.

111. **The Common Pump.** The common pump consists of a cylindrical barrel AB, furnished with a valve at B opening upwards, and with an air-tight piston, capable of being moved up and down by means of a rod.



The piston is also furnished with a valve opening upwards, and to the barrel at B is attached a pipe BC, called the suction pipe.

Let C be the surface of the water to be raised, and suppose the piston to be at B.

Let the piston be raised from B to A; the air in BC will, by virtue of its expansive force, press open the valve at B and fill the barrel AB, and in consequence of occupying a larger space ABC will exert a less pressure than before upon the surface of the water within the pipe. Hence, the pressure of the air within the pump being less than the atmospheric pressure upon the surface of the water, the water will be forced up the pipe BC until equilibrium be restored.

Let the piston now descend,—the valve at B will be closed, and the air in AB will escape through the valve in D. If the piston be again raised, the preceding process will be repeated, and the water will again rise in the pipe BC. In like manner upon every successive stroke of the piston the water will continue to rise in the pipe until, if BC be less than 32 feet, it at last reaches B.

Let the water be at B, and let the piston be pressed down to B. Then since the atmosphere presses upon the surface of the fluid at C, there will be transmitted by the fluid a pressure upon every square unit of the valve at B equal to $Æ - w.BC$ (Art. 109); and, consequently, when the piston is again raised, the water will force open the valve B, and will rise in the barrel as far as A, if AC be less than 32 feet, or to some point between B and A, if AC be greater than 32 feet.

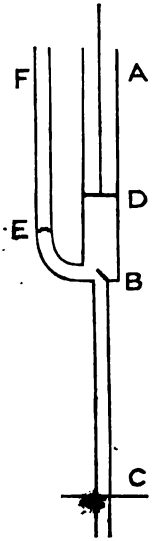
Upon the next descent of the piston, the valve in D will be forced open, and the water will flow through it, so that when the piston reaches B the water in the barrel will lie above the piston, and when the piston is raised will be raised along with it, and flow out at the spout A.

It will be seen that if B be more than 32 feet above C no water will rise into the barrel; and in such a case the pump will be useless. If B be less, but A more, than 32 feet, the water will only in part fill the barrel, and the discharge of the water during the ascent of the piston will not be continuous.

112. THE FORCING PUMP. The construction of the forcing pump is represented in the accompanying figure.

D is a piston with no valve, working in a barrel AB; BC is a pipe descending to the water, EF a tube leading upwards from B, at B and E are valves opening upwards.

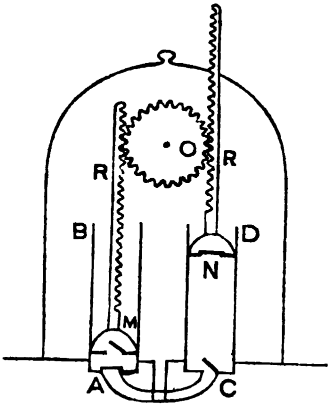
If the piston be raised from B to A, then, as in the common pump, the air in BC will press open the valve at B and fill the barrel, and the water will rise to a certain height in BC. As the piston descends the air in the barrel will be forced out through the valve at E; after a certain number of ascents the water will, if BC be less than 32 feet, pass into the barrel AB; and, upon its next descent, the piston will force the water in the barrel through the valve E into the pipe EF. As the piston continues to rise more water will pass into the barrel, and this, in its turn, will be forced by the descent of the piston into the pipe EF.



113. **The Double-Barrel Air Pump.** AB and CD are two cylindrical barrels connected together by the pipe AC, and communicating by means of another pipe with an air tight receiver R. M and N are pistons furnished with valves opening upwards, and are worked by the toothed wheel O, so that when M descends N will ascend, and *vice versâ*. At A and C are valves opening upwards.

Let M be at A, and N at D, and the wheel be turned so that N may descend and M ascend. As N descends the valve at C will close, and the air in the barrel CD will pass out through the valve in N. As M ascends, the pressure of the external air will close the valve in

M; and the pressure being removed from the upper side of the valve A, the air in the receiver will in consequence of its expansive force press open the valve and fill the barrel AB. The air originally in the receiver now occupying a larger space, viz. the receiver and barrel together, will be of diminished density.



If the wheel be again turned a similar process will take place, and the air in the receiver will again expand, so as to fill both receiver and barrel, and a second diminution of density will consequently take place.

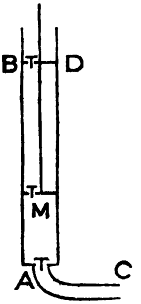
The process may be continued so long as the air in the receiver can by its expansive force press open the valves at A or C; but since the expansive force of the air is diminished with the diminution of its density, the action of the pump must, after a certain number of strokes, entirely cease.

114. THE SINGLE-BARREL AIR PUMP. This air pump consists of a single barrel AB, communicating with an air-tight receiver by the pipe C.

The piston passes through an air-tight collar in the plate BD. At B is a valve opening upwards.

The process of exhaustion is precisely similar to that described in Art. 113.

The effect of the valve at B is to relieve the piston from the pressure of the atmosphere, during a part of its ascent. For if the air in BM be of less density than atmospheric air, the pressure of the atmosphere will press down the valve at B, and will keep it closed until the piston by its ascent has so compressed the air in BM, that its density has become greater than that of atmospheric air.



The plate BD with its valve opening upwards is not peculiar to the single barrel air pump, but may, if required, be adopted in a double-barrel air pump. In fact, it is used in the more carefully constructed double-barrel pumps.

EXAMPLES.

1. What is the atmospheric pressure per inch when the barometer stands at 30 inches?

Ans. 14 lbs. 12 oz.

2. What change in the atmospheric pressure will be denoted by a change of 1 inch in the height of the barometric column? Ans. $7\frac{13}{15}$ oz.

3. If a barometer be formed of a liquid whose sp. gr. is 7·5, at what height will it stand when a quicksilver barometer stands at 30 inches? Ans. 54·4 in.

4. Will the siphon act if the longer leg be placed in the liquid to be drawn off?

5. What is the greatest available length of the shorter leg of a siphon used for drawing off water, the barometer standing at 30 inches? Ans. 34 ft.

6. What if the barometer stand at 29 inches?

Ans. 32 ft. $10\frac{2}{5}$ in.

7. What is the greatest available length of the

shorter leg of a siphon used for drawing off a liquid whose sp. gr. is 2·5; the barometer standing at 29½ inches? Ans. 13·37 ft.

8. If the spout of a common pump be 15 feet above the surface of the water to be raised, and the area of the piston be 10 square inches, what power will be required to work the pump, the handle being a lever whose longer arm is 30 inches, and shorter 3 inches? Ans. 6 lbs. 8⅛ oz.

9. If the spout of a common pump used for raising a liquid whose sp. gr. is 1·2 be 14 feet above the surface of the liquid, and the area of the piston be 8 square inches, what power will be required to work the pump, the handle being a lever whose longer arm is 3 feet and shorter 3 inches? Ans. 4 lbs. 13$\frac{7}{9}$ oz.

10. Find the sp. gr. of the heaviest liquid that can be drawn off by a siphon whose shortest leg is 64·4 inches, the barometer standing at 30 inches. Ans. 7·5.

CHAPTER VIII.

ON LIGHT.

115. *Light* is the name by which we describe the cause which produces the sensation of vision. Respecting its nature and mode of action nothing is certainly known. Different theories have been proposed with the view of explaining the various phenomena of light, but success in these attempts has hitherto been but partial.

116. A *self-luminous* body is one which by itself excites the sensation of vision. Bodies not self-luminous may excite the sensation of vision, but can do so only when some self-luminous body is present; such bodies simply transmit light which has originated elsewhere.

117. Any portion of space through which light is propagated is called a *medium*, whether the space be occupied by any body or not. Bodies through which light can be propagated are called *transparent;* those which entirely intercept it are called opaque. Strictly speaking, these terms do not distinguish different bodies from each other, but express qualities which to a greater or less degree belong alike to all bodies. There is no known substance so transparent as not to intercept some portion of light, nor on the other hand is any substance so opaque as not to transmit some light.

118. Experience teaches us that light, unless acted upon by some external cause, is propagated in straight lines. If, for instance, we take a hollow tube, and there be present no cause capable of changing the direction of light, we find that we can see through the tube only when straight lines can be drawn along it. If the tube be so bent that no straight line can be drawn along it, we cannot see through it.

119. A *ray* of light is the smallest conceivable portion of light, and is represented by the straight line along which it is supposed to be propagated. A collection of rays is called a *pencil* of light. A parallel pencil is one whose component rays are all parallel to each other, a converging pencil one in which the rays all proceed *to* a single point, and a diverging pencil one in which they all proceed *from* a single point. The point to or from which the rays proceed is called the *focus* of the pencil.

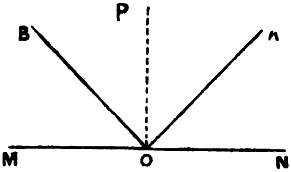
120. When light falls upon the surface of any body, one portion, greater or less, is turned back into the medium whence it came, or, as it is termed, *reflected;* another portion, greater or less, is transmitted through the new medium, and undergoes a change in its direction termed *refraction;* while a third portion, greater or less, is absorbed by the new medium.

121. When a ray of light falls upon any surface, the angle which it makes with the perpendicular to the surface at the point of incidence is termed the *angle of incidence;* the angle which the reflected ray makes with the same perpendicular is termed the *angle of reflection*, and similarly the angle which the refracted ray

makes with this perpendicular is termed the *angle of refraction.*

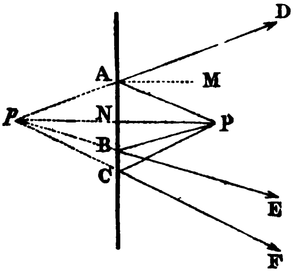
122. LAW OF REFLECTION. *The reflected ray lies in the same plane with the incident ray and the perpendicular to the surface at the point of incidence; and the angle of reflection is equal to the angle of incidence.*

Thus, if AO be a ray of light incident upon the reflecting surface MN, and if OB be the reflected ray, and OP the perpendicular to the surface, then will OB lie in the same plane with AO and OP, and the angle BOP be equal to the angle AOP.



123. *If a diverging pencil of light fall upon a plane reflecting surface, the focus of the reflected pencil will be at the same distance from the surface as that of the incident pencil, but on the opposite side of it.*

Let P be the focus of the incident pencil, and AC a section of the reflecting surface. Draw PN perpendicular to AC, and make *p*N = PN. Let PA be any incident ray, join *p*A, then shall *p*A produced, or AD be the direction of the reflected ray. For, by the construction, the triangles *p*NA PNA are equal, and therefore the angle A*p*N = the angle APN. But if AM be the perpendicular to the surface at A, then AM and P*p* being parallel, the angle

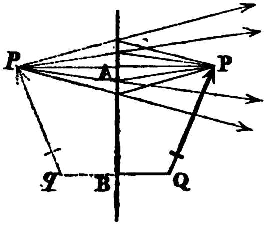


DAM = the angle ApN, and the angle PAM = the angle APN. Therefore the angle DAM = the angle PAM, or AD is the direction of the reflected ray. In a similar manner it may be shewn, that all the reflected rays will proceed from p, and therefore p is the focus of the reflected pencil.

124. From every point in a luminous body, whether self-luminous or illuminated, light is propagated in all possible directions. If then light from such a body fall upon any reflecting surface, or be transmitted through any refracting medium, each point may be regarded as the focus of an incident pencil. For every incident pencil there will then be a reflected or refracted pencil, each with its own focus. This set of points, made up of the foci of the reflected or refracted pencils, is termed the *image* of the luminous body. When the reflected or refracted rays actually pass through these points, the image is called *real*, but when they do not so pass the image is called *virtual*.

125. *To find the position and magnitude of the image when light from a luminous object falls upon a plane reflecting surface.*

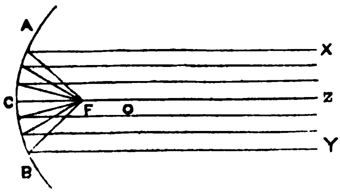
Let PQ be the given object, and AB a section of the reflecting surface. Draw Pp perpendicular to AB, and make pA = PA. Then, (Art. 123,) p is the focus of the reflected pencil, for all the light falling upon the surface from P. Similarly, if Qq be perpendicular to AB, and qB be equal to



QB, q will be the focus of the reflected pencil for all the light incident from Q. In like manner, for every incident pencil proceeding from points lying between P and Q, the focus of the reflected pencil will lie between p and q. Hence the image pq will be of the same size and shape as the object PQ. The image in this case is virtual, and is situated behind the surface in such a way that each point in it is at the same distance from the surface as the corresponding point in the object.

126. When the reflecting surface is plane all the rays of the reflected pencil, as already seen, diverge accurately from a single point. When the reflecting surface is spherical, the reflected rays do not in any case all proceed either to or from a single point. If, however, the incident pencil be small, the rays of the reflected pencil converge to, or diverge from a point so nearly, that their distance from it is inappreciable, and this point is regarded as a focus.

127. If a small pencil of parallel rays fall upon a spherical mirror, the focus of the reflected pencil is called the *principal focus* of the mirror. Thus, let XA, ZC, YB, be a parallel pencil falling upon the spherical mirror AB. Let F be the focus of the reflected pencil; F is the principal focus of the mirror. Let that ray which passes through O, the centre of the spherical surface, be regarded as the *axis* of the pencil. The point F will lie in the axis at a distance midway between O and the mirror. This distance of F from



the mirror is called the *focal length* of the mirror. Hence, *the focal length of any spherical mirror is equal to half the radius of the surface.*

128. *To find the focus of the reflected pencil when a small diverging pencil is incident upon a spherical mirror.*

Let u = the distance from the mirror of the focus of the incident pencil, v = the distance from the mirror of the focus of the reflected pencil, and f = the focal length of the mirror. Then, if that ray which passes through the centre of the surface be regarded as the axis of the incident pencil, the focus of the reflected pencil lies in this axis at a point whose distance from the mirror is determined by the following formula:—

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u},$$

in using which, distances measured on the side of the mirror opposite to that on which the focus of the incident pencil falls, are to be reckoned negative.

Ex. 1. The focus of a small pencil of light falling upon a concave spherical reflector, whose focal length is 5 inches, is at a point 12 inches from the mirror, to find the distance from the mirror of the focus of the reflected pencil.

Here $u = 12$, and $f = 5$; also f is positive, being measured on the same side of the mirror as u,

$$\therefore \quad \frac{1}{v} = \frac{1}{5} - \frac{1}{12} = \frac{7}{60},$$

$$\therefore \quad v = \frac{60}{7} = 8\tfrac{4}{7}$$

Ex. 2. The focal length of a concave spherical reflector is 6 inches, the focus of the incident pencil is 5 inches from the mirror, to find the focus of the reflected pencil.

Proceeding as before, we have

$$\frac{1}{v} = \frac{1}{6} - \frac{1}{5} = -\frac{1}{30},$$

$$\therefore \quad v = -30,$$

or the focus of the reflected pencil is at a distance 30 inches behind the mirror.

Ex. 3. The focal length of a convex spherical reflector is 8 inches, the focus of the incident pencil is 4 inches from the mirror, to find the focus of the reflected pencil.

Since the surface is convex, the focal length is not measured on the same side of the mirror as u, and consequently must be taken negatively; for f therefore we must write -8. Hence,

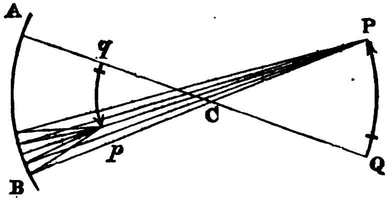
$$\frac{1}{v} = -\frac{1}{8} - \frac{1}{4} = -\frac{3}{8},$$

$$\therefore \quad v = -\frac{8}{3} = -2\frac{2}{3},$$

or the focus of the reflected pencil will be $2\frac{2}{3}$ inches behind the mirror.

129. *To find the position and magnitude of the image of an object placed before a concave spherical mirror.*

Let AB be a section of the mirror, and C the centre of the spherical surface. Let the object be the arc of a circle concentric with the mirror, or so small as not sensibly to differ from such an arc. Each point in the



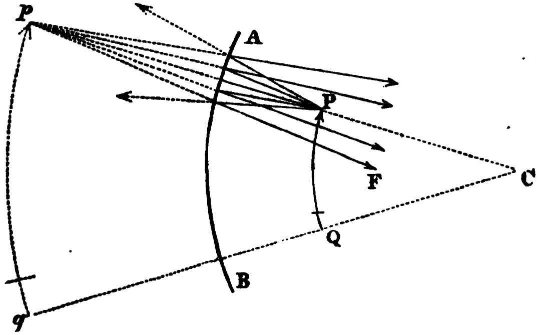
object being equi-distant from C, and therefore from the mirror; the foci of all the reflected pencils will be equi-distant from the mirror, and therefore from C also. The distance of the foci of the reflected pencils will be found from the formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}.$$

First; Let the object be placed beyond the centre of the spherical surface; that is, let u be greater than r. In this case, the value of v, as determined by the formula, is less than r, but greater than $\frac{1}{2}r$ or f, that is, the foci of the reflected pencils will lie between the principal focus and the centre. Draw PC, and let p be a point in this line, whose distance from the mirror equals the value of v, as already found; then a small pencil, incident upon the mirror from P, will be reflected to p. Similarly, a pencil from Q will be reflected to q; and, in like manner, for all pencils proceeding from the points in the object between P and Q, the foci of the reflected pencils will lie between p and q, pq therefore is the image of PQ. It will be seen that in this case the image is inverted, it will also always be found to be diminished. Since pq and PQ are corresponding arcs of different circles, they are as their respective radii; that is, the linear magnitudes of the image and object are as their respective distances from the centre.

Secondly; Let the object be placed between the centre and principal focus, that is, let u be less than r, but greater than f. In this case v will be greater than r. Let pq be the object, then, proceeding as before, the image will be a collection of points more distant from the mirror than C, as for instance at PQ. The image as before is inverted, but will be found always to be magnified.

Thirdly; Let the object be placed between the principal focus and the mirror, that is, let u be less than f. In this case v will be always negative, hence the foci of the reflected pencils, and therefore the image will lie behind the mirror. It will be seen from the figure, that the image is erect, and being more distant from the centre than the object is magnified.



When the foci of the reflected pencils are on the same side of the mirror as the foci of the incident pencils, that is, when v is positive, the light passes through the foci; and, in this case, the image is said to be *real*. When the foci of the reflected pencils are on the opposite side of the mirror, or v is negative, the light does not pass through the foci; in this case the image is said to be *virtual*.

130. *To find the position and magnitude of the image of an object placed before a convex spherical mirror.*

Let pq in the last figure denote the *object*. In the case of a convex mirror the value of v is always negative, and never greater than f. Hence the image will be at PQ on the opposite side of the mirror, and being nearer the centre than the object will be diminished.

131. Hence, for convex mirrors the image is always *virtual*, *erect*, and *diminished*.

For concave mirrors the image is *real*, *inverted*, and *diminished*, when the object is *beyond* the centre; *real*, *inverted*, and *magnified*, when the object is between the centre and principal focus; *virtual*, *erect*, and *magnified*, when the object is between the principal focus and the mirror.

Ex. 1. The focal length of a concave spherical mirror is 5 inches, to find the nature, position, and magnitude of the image of a small object placed 30 inches from the mirror.

The object being small, each point is supposed to lie at the same distance from the mirror, namely, 30 inches. Hence, to find the distance of image from mirror, we have

$$\frac{1}{v} = \frac{1}{5} - \frac{1}{30} = \frac{1}{6},$$

or $$v = 6.$$

Therefore v being positive, the image is real and inverted. It is 6 in. from the mirror, and consequently 4 in. from the centre. The image is diminished in proportion of 4 to 20, or 1 to 5.

Ex. 2. With the same mirror, to find the nature, position, and magnitude of the image, when the object is 4 inches from the mirror. In this case,

$$\frac{1}{v} = \frac{1}{5} - \frac{1}{4} = -\frac{1}{20},$$

or $$v = -20.$$

The image is 20 inches behind the mirror, is virtual and erect. The image is 30 inches from the centre, and the object 6 inches; the image therefore is magnified in the proportion of 30 to 6, or 5 to 1.

Ex. 3. The focal length of a convex spherical mirror is 6 inches, find the nature, position, and magnitude of the image, when the object is 3 inches from the mirror.

In this case the focal length is negative, therefore

$$\frac{1}{v} = -\frac{1}{6} - \frac{1}{3} = -\frac{1}{2},$$

or $$v = -2.$$

The image is 2 inches behind the mirror, virtual and erect. The image is 10 inches from the centre, and the object 15 inches; the image therefore is diminished in the proportion of 10 to 15, or 2 to 3.

EXAMPLES.

1. The focal length of a concave mirror is 10 inches, what is the magnitude of the image of a small object 30 inches from the mirror? Ans. one-half the object.

2. The focal length of a convex mirror is 8 inches, what is the position of the image of a small object 12 inches from the mirror?

Ans. $4\frac{4}{5}$ inches behind the mirror.

3. Shew that the linear magnitudes of the image and object are as their respective distances from the *mirror*.

4. At what distance from a concave mirror must an object be placed that the image may be exactly one-half the object? Ans. $3f$.

5. At what distance from a concave mirror must an object be placed that the image may be double the object? Ans. $\frac{3f}{2}$.

6. At what distance from a convex mirror must an object be placed that the image may be half the object?

Ans. f.

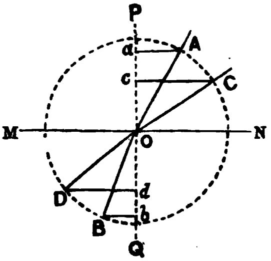
7. At what distance from a concave mirror must an object be placed that the image may be n times the object?

Ans. $\frac{(n+1)f}{n}$.

8. At what distance from a convex mirror must an object be placed that the image may be 1-n^{th} of the object?

Ans. $(n-1)f$.

132. **Law of Refraction.** The refracted ray lies in the same plane with the incident ray and the perpendicular to the surface at the point of incidence; and the angles of incidence and refraction are related in the following manner:—



Let AO be a ray incident upon MN, the surface of a refracting medium. Let OB be the refracted ray, and PQ the perpendicular to the surface drawn through O. With O as a centre, and any distance as a radius, describe a circle cutting the incident and refracted rays in A and B. From A and B draw Aa, Bb perpendicular to PQ, and let m represent the ratio of Aa to Bb, that is, let

$$m = \frac{Aa}{Bb}.$$

Then, if CO represent any other ray incident upon the surface MN at O, and OD be the direction of this ray after refraction, and if Cc, Dd be the perpendiculars to

PQ, the ratio of Cc to Dd will also be m, that is, the refracted ray will make with PQ such an angle that,

$$\frac{\mathrm{C}c}{\mathrm{D}d} = m.$$

133. The quantity m is termed the *refractive index*, and the law of refraction affirms, that for any two given media the value of m remains the same, however the angle of incidence may vary.

When light passes from a rarer into a denser medium, the value of m is greater than unity, and consequently the angle of refraction is less than the angle of incidence. When light passes from a denser into a rarer medium m is less than unity, and the angle of refraction greater than the angle of incidence. Hence, by refraction, a ray of light is bent *towards* the perpendicular when passing into a denser medium, and *from* the perpendicular when passing into a rarer medium.

Between atmospheric air and plate or crown glass, $m = \frac{3}{2}$ nearly; between atmospheric air and water, $m = \frac{4}{3}$. When the light passes from glass or water into air, the refractive indices are the reciprocals of the values just given, and are therefore $\frac{2}{3}$ and $\frac{3}{4}$ respectively.

134. When a small pencil of light is incident upon the plane surface of a medium, the distance of the focus of the refracted pencil from the surface is m times the distance of the focus of the incident pencil. When light passes from a rarer into a denser medium m is greater than unity, and consequently in this case the focus of the refracted pencil is more distant from the surface than the focus of the incident pencil. When light passes from a denser into a rarer medium m is

less than unity, and consequently the focus of the refracted pencil is nearer the surface than the focus of the incident pencil. Hence, when a luminous body is seen by light which has been refracted at the plane surface of a denser medium, each point in it will appear more distant from the surface in the proportion of m to 1; but when seen by light refracted at the plane surface of a rarer medium, each point will appear nearer to the surface than it really is, in a similar proportion. Thus, for example, the bottom of a vessel 12 inches deep when filled with water will appear only $12 \times \frac{3}{4}$, or 9 inches distant from the surface; and a piece of glass 6 inches thick will appear only $6 \times \frac{2}{3}$, or 4 inches thick.

135. Any portion of a refracting medium which has two opposite surfaces, either both spherical, or one spherical and the other plane, is termed a *lens*.

Lenses are of two classes, convex and concave. *Convex* lenses are those which are thickest at the centre. *Concave* lenses those which are thinnest at the centre.

Of convex lenses those which have both surfaces convex are called *double convex* lenses; those which have one surface spherical and the other plane are called *plano-convex* lenses; and those which have one surface convex and the other concave (the radius of the CONCAVE surface being the greater) are called *meniscus lenses*. And, similarly, concave lenses are *double concave* when both surfaces are concave; *plano-concave* when one surface is spherical and the other plane; *convexo-concave* when one surface is convex and the other concave, the radius of the CONVEX surface being the greater.

136. The *axis* of a lens is the line joining the centres of the two surfaces when both are spherical, but when one of the surfaces is a plane, is the perpendicular to the plane through the centre of the spherical surface. When the thickness of a lens is so small that it may be disregarded, the *centre* of the lens is the point where the axis meets the lens.

137. When rays of light diverging from a luminous point fall upon a lens, the refracted rays will be less divergent than the incident rays if the lens be convex, but more divergent if the lens be concave. A pencil of divergent rays *may* therefore become convergent after refraction by a convex lens, but cannot become so after refraction by a concave lens.

When rays of light diverging from a luminous point fall upon any spherical lens, the refracted rays do not in any case proceed exactly to or from a single point. Those rays, however, which are near the axis will, after refraction, proceed to or from a certain point so nearly, that their distance from it is not appreciable; this point is therefore regarded as a focus.

138. When a small parallel pencil is incident upon a lens, the focus of the refracted pencil is called the *principal focus*, and the distance of this point from the lens is called its focal length. The position of the principal focus of a lens is determined by the following formula:—

$$\frac{1}{f} = (m - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$

Where f is the focal length, m the refractive index, r the radius of the first surface of the lens, and s the radius of the second surface, lines being considered positive when on the same side of the lens as that whence the light proceeds, and negative when on the opposite. When the value of f is *negative*, the refracted rays pass *to* the focus, but when positive they proceed from it.

Ex. 1. To find the principal focus of a double convex glass lens, when the radii of both surfaces are equal. Here $m = \frac{3}{2}$ and $s = r$. The radius of the first surface is negative, and that of the second positive. Hence,

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(-\frac{1}{r} - \frac{1}{r}\right)$$
$$= -\frac{1}{r},$$
$$\text{or} \quad f = -r.$$

That is, the focus is negative, and the focal length of the lens is equal to the radius of the surface.

Ex. 2. To find the principal focus of a plano-concave glass lens.

Here, as before, $m = \frac{3}{2}$. The radius of the first surface is positive. The second surface being plane may be regarded as a sphere of infinite radius. But if s be infinite $\frac{1}{s} = 0$. Therefore, in this case,

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{r} - 0\right)$$
$$= \frac{1}{2r},$$
$$\therefore \quad f = 2r,$$

or the focal length is twice the radius of the concave surface.

By proceeding in a similar way through the different kinds of lenses, it will be found that the principal focus of all convex lenses is negative, or the refracted pencil converges to the focus, and that the principal focus of all concave lenses is positive, or the refracted pencil diverges from the focus.

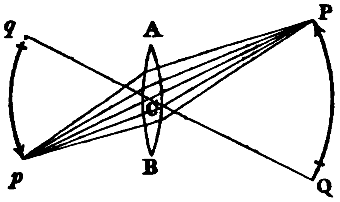
139. When a small diverging pencil of light is incident upon a lens, the position of the focus of the refracted pencil is determined by the following formula:—

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u},$$

where f stands for the focal length of the lens, u for the distance of the focus of the incident pencil from the lens, and v for the distance of the focus of the refracted pencil from the lens. If the focus of the incident pencil be in the axis of the lens, the focus of the refracted pencil will also be in the axis. When the focus of the incident pencil does not lie in the axis of the lens, the focus of the refracted pencil will lie in the line joining the focus of the incident pencil with the centre of the lens. This, however, is applicable only to the cases in which the distance of the focus of the incident pencil from the axis is small when compared with its distance from the lens. Such cases are alone considered in the present chapter. Let P (fig. Art. 141) be the focus of a small pencil incident upon the lens AB, whose centre is at C, then p the focus of the refracted pencil will lie in the line PC at a distance pC as determined by the formula given above.

140. *To determine the nature, position, and magnitude of the image of a small object placed before a convex lens.*

Let AB be the lens, whose focal length is 4 inches, and PQ the object whose distance from the lens is 12 inches.



Since the lens is convex, the focal length is negative (Art 138), therefore in this case $f = -4$, and for the pencil proceeding from any point in the object u will be 12, therefore by the formula

$$\frac{1}{v} = -\frac{1}{4} + \frac{1}{12} = -\frac{1}{6},$$

$$\therefore \qquad v = -6.$$

Hence, for the pencil proceeding from P, join PC, and in PC produced take pC = 6, then p will be the focus of the refracted pencil. Similarly q will be the focus for the pencil proceeding from Q, and for all pencils proceeding from points between P and Q, the foci of the refracted pencils will lie between p and q. Therefore pq is the image of the object PQ. Since pq and PQ are similar arcs of different circles they are as the radii. Therefore the image : the object :: pc : PC, that is, as 6 : 12, or as 1 : 2. The image it will be seen is inverted, and not being on the same side of the lens as the object, is *real.*

141. By proceeding similarly with objects placed at different distances from the lens, it will be found:—

1. That for objects placed at a greater distance than twice the focal length the image is real, inverted, and diminished.

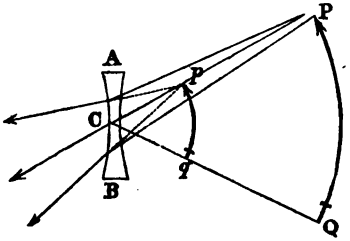
2. That for objects placed at twice the focal length the image is real, inverted, and of the same size as the object.

3. That for objects distant less than twice the focal

length, but more than the focal length, the image is real, inverted, and magnified.

4. That for objects distant less than the focal length the image is virtual, erect, and magnified.

142. *To determine the nature, position, and magnitude of the image of a small object placed before a concave lens.*



Let AB be the lens, and its focal length 5 inches. Let PQ be the object, distant 8 in. from the lens. The lens being concave the focal length is positive (Art. 138). Therefore for determining the foci of the refracted pencil the formula gives

$$\frac{1}{v} = \frac{1}{5} + \frac{1}{8} = \frac{13}{40},$$

$$\therefore \qquad v = 3\tfrac{1}{13}.$$

Hence, for the pencil proceeding from P, join PC, and take $pC = 3\frac{1}{13}$, then p is the focus of the refracted pencil. Similarly, q will be the focus for the pencil proceeding from Q, and for all pencils proceeding from points between P and Q, the foci of the refracted pencils will lie between p and q. Hence the image pq is virtual, erect, and diminished, the image being to the object as $3\frac{1}{13}$ to 8.

143. A similar result will be obtained whatever the distance of the object from the lens. With concave lenses the image is always virtual, erect, and diminished.

EXAMPLES.

1. Find the focal length of the meniscus lens, the radii of whose surfaces are 4 and 5 inches.

Ans. 40 inches.

2. Find the focal length of a double concave lens, the radius of each surface being 3 inches.

Ans. 3 inches.

3. Find the focal length of a double convex lens, formed of a substance whose refractive index is $\frac{8}{5}$, the radius of each surface being 5 inches.

Ans. $4\frac{1}{6}$ inches.

4. If the radius of the spherical surface of a plano-concave lens be 6 inches, what must be the refractive index of the material of the lens, in order that its focal length may be 10 inches? Ans. $\frac{8}{5}$.

5. If an object be placed 10 inches from a convex lens, whose focal length is 6 inches, find the position of the image. Ans. 15 inches behind the lens.

6. If an object be placed 10 inches from a convex lens, whose focal length is 12 inches, find the position of the image. Ans. 5 feet in front of the lens.

7. How far in front of the convex lens, whose focal length is 1 inch, must an object be placed in order that the image may fall 6 inches behind the lens?

Ans. $1\frac{1}{5}$ in.

8. How far from the concave lens, whose focal length is 3 inches, must an object be placed in order that the image may be 2 inches from the lens. Ans. 6 inches.

CHAPTER IX.

ON THE NATURE OF SOUND.

144. When the particles of an elastic body are suddenly displaced at any point, a peculiar motion termed *vibration* is communicated to all the particles of that body. The displaced particle oscillates or vibrates through small distances about its position of equilibrium, its excursions on either side gradually diminishing, and at length altogether ceasing. A similar vibratory movement is communicated to the other particles of the body. The velocity with which a vibration is transmitted from one point in the body to another is different in different bodies, and must carefully be distinguished from the absolute velocity of the particles themselves.

An elastic body in a state of vibration, may communicate a similar motion to the particles of another elastic body in contact with it.

145. *The sensation of sound is produced by the vibration of an elastic body in contact with the ear.*

If air be the medium in contact with the ear, as is most commonly the case, sound is produced by the vibrations of the air; if water be the medium in contact with the ear, the vibrations of the water produce the sensation; if an iron or glass rod, then the vibrations of the particles of the rod produce it.

The vibratory motion which produces the sensation of sound is either produced directly in the medium in contact with the ear, or is communicated to that medium by the vibration of another elastic body. When a bell is struck, a vibratory motion is given to the particles of the bell, and we hear the sound, because vibrations are thence communicated to the surrounding air, which is the medium in contact with our ear. If, however, the bell be so placed, that its vibrations cannot be communicated to the air, no sound will be heard, however violently it may be rung. For instance, if the bell be suspended under the receiver of an air pump, and be made to ring, as rarefaction proceeds the sound becomes gradually weakened, and at length becomes inaudible.

146. THE VELOCITY OF SOUND. The average velocity of the propagation of sound in atmospheric air is found by experiment to be about 1130 feet in a second. In water the velocity of sound is 4900 feet in a second, and along a deal rod the velocity is as great as 17,000 feet in a second.

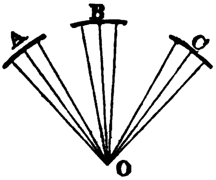
Increase of temperature increases the velocity of sound. According to the best experiments, the velocity of sound in air at 62° Fahrenheit, is 1125 feet in a second. Every increase or decrease of temperature of 1° Fahrenheit, causes a corresponding increase or decrease of 1·14 feet in the velocity of sound. Accordingly the velocity of sound in air at the freezing point will be 1090 feet per second.

In any given medium, the velocity of propagation is found to be the same for all sounds, whatever their loudness or tone.

147. Musical Tones. If a succession of vibrations performed in different times strike upon the ear, the sound produced is called *noise*, but if a succession of vibrations performed in equal times strike upon the ear, an effect pleasing to the ear is produced, and the sound is distinguished as a *tone*. It is found, however, that to produce this effect, there must be at least 30 double vibrations in a second.

Tones differ according to the number of vibrations performed in a second. That sound is called the higher which is caused by the greater number of vibrations. The note represented by the middle A of the treble clef requires 420 double vibrations per second. The note C, next below this, is the effect of 252 double vibrations per second.

148. Echoes. Sound, when it meets with any hard or elastic surface is reflected, and, as in the reflection of light, the angles of incidence and reflection are equal. Hence it may happen, that a sound meeting with one or more of such surfaces is reflected back to the point whence it originated. Thus, let O be an origin of sound, and let ABC be portions of a spherical surface of which O is the centre; then all the sound which reaches A, B, and C from O is reflected back to O, and, if the quantity of sound be sufficient, will be heard by a person placed at O. It is in this way echoes are formed, an echo being sound heard by reflection. In



order to the existence of an echo, three conditions are necessary:—

1. That a sufficient quantity of sound be reflected to the ear to produce sensation.

2. That all the sound reflected from different points of the reflecting surface reaches the hearer at the same time. If part of the reflecting surface, C for instance, were more distant from O than A, the sound falling on both surfaces might still be reflected to O, but the sound from C would reach O later than that from A, and the result would be a confused, and not a distinct sensation.

3. That the interval between the origination of the sound and its return to the origin be sufficiently great to allow that the production of the sound should cease before any of the reflected sound reaches the hearer. If the space over which the sound travels in passing to and from the origin be considerable, time may be given for the utterance of a series of sounds before any of the reflected sound reaches the hearer; and, in such a case, several words may be uttered and will be distinctly echoed. If the distance be short, there may be time for only a few syllables, or even for a single syllable. If the interval be so short as not to allow of the complete production of a sound before the reflection reaches the hearer, no distinct echo can of course be heard.

It is not necessary that the reflecting surfaces A, B, C be accurately spherical, for since a small portion of a spherical surface does not sensibly differ from a plane, a number of plane surfaces situated at equal distances from O, may serve for the production of an echo.

MISCELLANEOUS EXAMPLES.

1. If two forces, 20 and 15, act upon a point at an angle of 60°, what is the magnitude of their resultant? Ans. 30·4.

2. If two forces, 10 and 12, act upon a point at an angle of 45°, what is the magnitude of their resultant? Ans. 20·34 nearly.

3. If two forces, 20 and 10, act upon a point at an angle of 30°, what is the magnitude of their resultant? Ans. 29·1 nearly.

4. If a bar, whose length is 20 inches and weight 16 ounces, be used as a lever of the first kind, find the distance of the fulcrum from the weight when a power 4 ounces sustains a weight 60 ounces. Ans. 3 inches.

5. Find the pressure upon the surface of a globe 12 inches in diameter when immersed in water so as to be just covered.* Ans. 98 lbs. $3\frac{3}{7}$ oz.

6. A weight of 9 lbs. descends, drawing up another of 7 lbs. over a fixed pulley; find the space described in 5 seconds. Ans. 50 feet.

7. What horizontal force will sustain a weight of 240 lbs. on an inclined plane rising 5 in 13? Ans. 100 lbs.

8. Three parallel forces, 6, 8, and 10, act upon a

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* The surface of a globe is four times the area of a circle of corresponding radius.

body at distances 10, 12, and 14 respectively from a point without them; what is the distance of their resultant from the same point? Ans. $12\frac{1}{3}$.

9. If the arms of a bent lever of uniform thickness and density be 8 and 10 inches respectively, and the distance between their bisections be 6 inches, find the distance of the centre of gravity of the lever from the bisection of the shorter arm. Ans. $3\frac{1}{3}$ in.

10. If a cubic vessel whose edge is 12 inches be filled with a liquid whose specific gravity is 2·368, what is the pressure upon a side of the vessel?

Ans. 74 lbs.

11. If the focal length of a double convex lens is 5 inches, the radius of each surface being 8 inches, what is the refractive index of the material of the lens?

Ans. $\frac{9}{5}$.

12. The pressure upon a surface containing 54 square inches is 125 lbs.; what is the depth of its centre of gravity below the surface of the water?

Ans. 5 ft. 4 in.

13. If a cord 18 inches in length be fastened at two points in the same horizontal line, distant 12 inches apart, and if a smooth ring upon the cord sustain a weight of 20 lbs., what is the tension in the cord?

Ans. 13·4 lbs.

14. A mixture of gold and silver weighs 20 ounces, and its specific gravity is 12; what is the quantity of gold in the mixture? Ans. 5·48 oz.

15. The pressure upon a surface containing 20 square inches, at a depth of 12 inches, is 15 lbs.; what is the specific gravity of the fluid? Ans. 1·728.

16. Two convex lenses are placed 10 inches apart; the focal length of the first is 3 inches, and that of the second 5 inches; an object is placed 5 inches in front

of the first lens; find the position of the image formed by the second lens.

Ans. 5 in. in front of the second lens.

17. If two equal bars, of uniform thickness and density, be used in combination as levers of the first kind, and if the length of each of the larger arms $= a$, and of each of the shorter arms $= b$, and the weight of each bar $= w$, shew that, when P and W are in equilibrium, $Pa^2 + \frac{1}{2} w (a^2 - b^2) = W b^2$.

18. What must be the diameter of a screw, the distance between the threads being $\frac{1}{2}$ inch, in order that the mechanical advantage may be 44. Ans. 7 in.

19. With what velocity must a body be projected vertically upwards that it may rise 225 feet?

Ans. 120 ft.

20. If a bar whose length is $2l$ and weight W′ be used as a lever of the first kind, find the distance of the fulcrum from the centre of the bar when a power P sustains a weight W. Ans. $l \frac{W - P}{P + W + W'}$.

21. If three pullies whose weights are w_1, w_2, and w_3 be arranged according to the first system, and P and W be in equilibrium, shew that if P descend through a space a, and x_1, x_2, x_3 be the spaces through which the pulleys severally rise in consequence, then

$$Pa = Wx_1 + w_1 x_1 + w_2 x_2 + w_3 x_3.$$

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